

the Bain deformation. Twinned martensite in Fe-7Al-2C and Fe-8Al-2C wt% alloys tend to exhibit an abnormally large  $c/a$  ratio because of the coherent precipitation of fine  $\kappa$ -carbide (Section 11.7) in the austenite; this coherency is inherited by the martensite. The precipitates themselves undergo the Bain deformation leaving their large density of carbon atoms along the  $c$ -axis. Other contributions to tetragonality come from specific carbon atom configurations in the austenite when the alloy contains a large aluminium concentration [190]. These explanations seem to fail for Fe-20Ni-1C wt% and similar alloys, where an anomalously large tetragonality is observed [188, 189]; recent first-principles calculations suggest instead that the cause of the anomaly is that the nickel makes the alloy more elastically compliant than Fe-C [186].

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## Notes

<sup>1</sup>The  $\gamma \rightarrow \epsilon$  transformation and martensitic transformation in ordered  $\text{Fe}_3\text{Be}$  (simple shearing) [191] are examples where the transformation strain is an invariant-plane strain.

<sup>2</sup>The assumption of the exact KS orientation means that there was no invariant line in the interface, and the martensite modelled was not a three-dimensionally enclosed plate.

<sup>3</sup>Notice that a combination of two non-coplanar invariant-plane strains gives an invariant-line strain, the invariant-line lying at the intersection of the two invariant-planes.

<sup>4</sup>This would be the invariant line in the  $\alpha'/\gamma$  interface. The interfacial dislocations would have their line vectors parallel to this invariant line.

<sup>5</sup>This is evident from Figure 5.8b where there are two different undistorted lines in the plane of the diagram.

<sup>6</sup>The matrices can be converted from the basis  $F$  to  $\gamma$  using a similarity transformation. Since  $\mathbf{f}_i \parallel \mathbf{a}_i$  we find that  $(F S F) = (\gamma S \gamma)$ .

<sup>7</sup>The actual magnitude is dependent on the exact thermodynamic model used for the calculations. This is particularly the case with martensite since the equilibrium data measured at high temperatures have to be extrapolated to much lower temperatures. Any analysis should therefore be internally consistent in the model and data used, but this has not always been the case in the published literature.

<sup>8</sup>We have not considered this term before. It comes from the Bowles and Mackenzie theory where a small uniform dilatation is in principle permitted. Experiments to date have not revealed such a dilation.

<sup>9</sup>Similar observations have been reported for martensitic transformation in TiC, Chapter 11.

<sup>10</sup>The fault energy would in fact have to be negative in order for the partial dislocations to overcome any lattice friction, such as that arising from the Peierls barrier to dislocation motion.

<sup>11</sup>A cubic structure will be mechanically stable if  $C_{11} - C_{12} > 0$ ,  $C_{11} + 2C_{12} > 0$  and  $C_{44} > 0$ .

<sup>12</sup>This statement, which is commonly made, is true only when the observations are made with an ordinary time resolution, i.e. no better than about a millisecond. More precise measurements must obviously reveal that athermal martensite takes time to form. Thus, Thadhani and Meyers [138] found that martensite normally considered to be athermal exhibits isothermal character in a microsecond regime.

<sup>13</sup>Not all martensite forms in this way; the morphology of lath martensite is in the form of packets of parallel plates. There do not appear to be any theoretical treatments for the kinetics of lath martensite.

<sup>14</sup>Autocatalysis refers to the case where the formation of one plate stimulates the nucleation of many others. This can lead to a sudden burst of transformation. The cascade of transformation usually is limited to an individual grain of austenite.

<sup>15</sup>This is analogous to the explanation of Hall and Petch for the decrease in yield strength at the grain size becomes coarser.