

- 7.37 (c) Choose  $N$  such that  $P(Y > N) < 0.5 \times 10^{-3}$ , where  $Y$  is a Poisson variable with parameter  $9 \times 0.8 = 7.2$ . This gives  $N = 17$ . (d)

$$\sqrt{X^2} = (1)(2) = 2$$

## Solutions for Chapter 8

8.3 (a) 0.013.

$$(b) f_{X|Y}(x|0) = \frac{1}{\pi} e^{-x^2/2}, \text{ for } -\infty < x < \infty.$$

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- 8.5 (a) For the joint density of  $B_1$  and  $B_2$ , since

$$f_{B_1 B_2}(x_1, x_2) = \left( \frac{1}{2\pi} e^{-x_1^2/2} \right) \left( \frac{1}{2\pi} e^{-x_2^2/2} \right) = \frac{1}{4\pi^2} e^{-(x_1^2 + x_2^2)/2},$$

it follows that

$$f_{B_1 B_2}(x_1, x_2) = f_{B_1}(x_1) f_{B_2}(x_2) = \left( \frac{1}{\sqrt{2\pi}} e^{-x_1^2/2} \right) \left( \frac{1}{\sqrt{2\pi}} e^{-x_2^2/2} \right) = \frac{1}{2\pi} e^{-(x_1^2 + x_2^2)/2}.$$

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