BIBLIOGRAPHY

(The number of each note is followed by the number of the page on which reference is made to it)

CHAPTER I

[1] (6) See e.g. Weyl, Raum Zeit Materie, 5th ed., Berlin 1923, p. 15.

[2] (13) Cf. D. Hilbert, Grundlagen der Geometrie, 7th ed., Leipzig 1930, Chapter 7.

[3] (14) We mention in particular: W. Burnside, Theory of Groups of Finite Order, 2nd ed., Cambridge 1911; G. A. Miller, H. F. Blichfeldt, L. E. Dickson, Theory and Application of Finite Groups, New York 1916; A. Speiser, Theorie der Gruppen von endlicher Ordnung, 3rd ed., Berlin 1937; H. Zassenhaus, Lehrbuch der Gruppentheorie I, Berlin 1937; the chapter on groups in v. d. Waerden, Moderne Algebra I, 2nd ed., Berlin 1937; and Chapter III, §§1-3, in Weyl, Gruppentheorie und Quantenmechanik, 2nd ed., Leipzig 1931.

[4] (14) Vergleichende Betrachtungen über neuere geometrische Forschungen, Erlangen 1872; also Math. Ann. *43*, 1893, p. 63, and Gesammelte mathematische Abhandlungen I, Berlin 1921, p. 460.

[5] (15) G. W. Leibniz, Initia rerum Mathematicarum metaphysica, in Leibnizens Mathematische Schriften, ed. C. J. Gerhardt, VII, Berlin 1848-63, p. 17; Zur Analysis der Lage, ibid. V, p. 178. How clearly Leibniz saw the problem of relativity is shown by his correspondence with Clarke, in particular his 3rd letter, Nos. 4 and 5, and his 5th letter, No. 47 (easily accessible in G. W. Leibniz, Philosophische Werke, ed. A. Buchenau and E. Cassirer, I, 2nd ed., in Meiner's Philosophische Bibliothek, Leipzig 1924).

[6] (17) Cf. Weyl, Raum Zeit Materie, 5th ed., Berlin 1923, p. 16.

[7] (18) For this notion and the foundations of the theory of invariants cf. B. L. v. d. Waerden, Math. Ann. *113*, 1936, p. 14.

[8] (22) I. Kant, Prolegomena zu einer jeden künftigen Metaphysik, die als Wissenschaft wird auftreten können, Kant's Werke, ed. Preuss. Akad. d. Wissensch., IV, Berlin 1903, p. 286.

CHAPTER II

[1] (27) Jour. reine angew. Math. *30*, 1846, p. 1. Coll. Math. Papers I, Cambridge 1889, p. 117.

[2] (27) Phil. Transact., vols. 144, 145, 146, 148, 149, 151, 157, 159, 169 (1854-1878). Coll. Math. Papers, vol. II, Nos. 139, 141, 144, 155, 156, 158; vol. IV, No. 269; vol. VI, No. 405; vol. VII, No. 462; vol. X, No. 693. The text refers to the first six numbers.

[3] (27) Math. Ann. *36*, 1890, p. 473; *42*, 1892, p. 313.

[4] (28) E. Galois, Oeuvres, Paris 1897, in particular his letter to Aug. Chevalier written on the eve of his death.

[5] (29) A modern book on the subject that will appeal to the mathematical reader is P. Niggli's Geometrische Kristallographie des Discontinuums, Leipzig 1919. A. Speiser's book, cited Chap. I[3], contains two interesting chapters on crystallographic classes and the symmetries of ornaments.

[6] (29) Lie himself synthesized his theories in the big three-volume work: S. Lie and F. Engel, Theorie der Transformationsgruppen, Leipzig 1893.

[7] (29) All of Frobenius' papers were published in the Sitzungsber. Preuss. Akad. A complete list of titles is to be found in Speiser's book, p. 143.

[8] (29) Thèse, Paris 1894, which is dependent on Killing's earlier but incomplete work, Math. Ann. *31*, *33*, *34*, *36* (1888-1890). E. Cartan, Bull. Soc. Math. de France *41*, 1913, p. 53.

[9] (29) W. R. Hamilton, Lectures on Quaternions, Dublin 1853. B. Peirce, Linear Associative Algebra, Washington 1870, and Am. Jour. of Math. *4*, 1881, p. 97. Th. Molien, Math. Ann. *41*, 1893, p. 83; *42*, 1893, p. 308.

[10] (29) J. H. M. Wedderburn, On Hypercomplex Numbers, Proc. London Math. Soc. (2) *6*, 1908, p. 77.

[11] (29) I. Schur, Trans. Amer. Math. Soc. (2) *15*, 1909, p. 159.

[12] (29) For the history of mathematics in the nineteenth century cf. F. Klein, Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert I, Berlin 1926.

[13] (39) A. Capelli, Math. Ann. *29*, 1887, pp. 331.

[14] (47) H. Weyl, Math. Zeitschr. *20*, 1924, p. 139.

[15] (51) R. Weitzenböck, Komplex-Symbolik, Leipzig 1908; Invariantentheorie, Groningen 1923, III. Abschnitt.

[16] (52) The first main theorem for orthogonal vector invariants was first proved by E. Study, Ber. Sächs. Akad. Wissensch. 1897, p. 443. The following treatment according to Weyl, Math. Zeitschr. *20*, 1924, p. 136.

[17] (56) The group of Euclidean movements, i.e. the n-dimensional orthogonal group extended by a rim of width ν, and its vector invariants were treated by R. Weitzenböck. See his Invariantentheorie, Groningen 1923, XII. Abschnitt; also Wanner, Dissertation Zürich 1926, and the Author's mimeographed Notes on Elementary Theory of Invariants, Princeton 1935-36.

[18] (56) Cayley, Jour. reine angew. Math. *32*, 1846, = Coll. Math. Papers I, No. 52, p. 332. About the algebraic structure of the orthogonal group and the other classical groups in a field either of characteristic zero or of prime characteristic, consult v. d. Waerden, Gruppen von linearen Transformationen, Ergebn. d. Math. *4*, 2, Berlin 1935, and the literature cited there. The systematic study of the groups mod p was undertaken by L. E. Dickson; see his book on Linear Groups, Leipzig 1901.

[19] (66) Cf. E. Witt, Jour. reine angew. Math. *176*, 1937, p. 31; Satz 2.

[20] (70) E. Pascal, Mem. Accad. dei Lincei (4) *5*, 1888. B. L. v. d. Waerden, Math. Ann. *95*, 1926, p. 706.

CHAPTER III

[1] (81) I. Schur in his "Neue Begründung der Theorie der Gruppencharaktere," Sitzungsber. Preuss. Akad. 1905, p. 406.

[2] (87) The treatment here follows a paper by the author in Annals of Math. *37*, 1936, section 1, pp. 710–718. For the abstract approach see Deuring, Algebren, Ergebn. Math. *4*, 1, Berlin 1935, in particular Chapters I–IV. At the basis of the whole modern development of the theory of associative algebras is Wedderburn's paper Chap. II[10]. L. E. Dickson's books: Linear Algebras, Cambridge Tracts *16*, 1914; Algebras and Their Arithmetics, Chicago 1923, and its revised German edition "Algebren und ihre Zahlentheorie," Zürich 1927, are landmarks of the development.

[3] (92) Burnside, Proc. London Math. Soc. (2) *3*, 1905, p. 430. G. Frobenius and I. Schur, Sitzungsber. Preuss. Akad. 1906, p. 209.

[4] (99) Cf. Chapter V of my book Gruppentheorie und Quantenmechanik, 2nd ed., Leipzig 1931.

[5] (101) Whence follows the full reducibility of any of its representations. This fundamental fact was first proved by H. Maschke, Math. Ann. *52*, 1899, p. 363.

[6] (106) Cf. H. Weyl, Duke Math. Jour. *3*, 1937, p. 200.

[7] (111) In its application to the algebra of all bisymmetric transformations in tensor space our method (II) is closely related to the very first treatment of the representations of the full linear group by I. Schur in his Dissertation, Ueber eine Klasse von Matrizen, die sich einer gegebenen Matrix zuordnen lassen, Berlin 1901, while method (I) is applied to this problem in: I. Schur, Sitzungsber. Preuss. Akad. 1927, p. 58. Cf. van der Waerden, Math. Ann. *104*, 1931, pp. 92 and 800.

[8] (112) $a \to a^J$ is an involutorial anti-automorphism operating on the elements a of an algebra if

$$(a + b)^J = a^J + b^J, \qquad (\lambda a)^J = \lambda \cdot a^J, \qquad (ab)^J = b^J \cdot a^J; \qquad (a^J)^J = a \qquad \{\lambda \text{ any number}\}.$$

Our roof operation is of this type. Algebras with an involutorial anti-automorphism have been thoroughly studied by A. A. Albert; see in particular his paper, Ann. of Math. *36*, 1935, p. 886. They are important for the theory of the so-called Riemann matrices, cf. Albert, l.c., and Weyl, Ann. of Math. *37*, 1936, p. 709, and moreover for the algebraic construction of Lie algebras, cf. Ch. VIII[9].

[9] (114) Semi-linear transformations were first introduced by C. Segre, Atti Torino *25*, 1889, p. 276. In the field $K^\dagger$ of complex numbers one has the automorphism consisting in the transition to the conjugate-complex ("antilinear transformations"). The theory of representations of a finite group by semi-linear transformations was given by T. Nakayama and K. Shoda, Jap. Jour. of Math. *12*, 1936, p. 109. For the corresponding generalization of our theory see Weyl l.c.[6]; for semi-linear or anti-linear transformations in general: J. Haantjes, Math. Ann. *112*, 1925, p. 98; *114*, 1937, p. 293; N. Jacobson, Ann. of Math. *38*, 1937, p. 485; Asano and Nakayama, Math. Ann. *115*, 1937, p. 87; Nakayama, Proc. Phys. Math. Soc. Japan *19*, 1937.

CHAPTER IV

[1] (115) l.c., Ch. III[1].

[2] (120) A. Young, Proc. London Math. Soc. (1), *33*, 1900, p. 97; (1) *34*, 1902, p. 361. G. Frobenius, Sitzungsber. Preuss. Akad. 1903, p. 328. v. Neumann's simplified procedure in: van der Waerden, Moderne Algebra II, Berlin, 1931, §127.

[3] (127) A particularly neat way of carrying out such a construction was indicated by W. Specht, Math. Zeitschr. *39*, 1935, p. 696; see especially sections IV and V of his paper. It is based on I. Schur's earlier work, specifically Sitzungsber. Preuss. Akad. 1908, p. 64. The same goal is attained in A. Young's series of publications in the Proc. London Math. Soc. "On the Quantitative Substitutional Analysis" starting with those cited under[2] and followed by (2) *28*, 1928, p. 285; (2) *31*, 1930, p. 253; (2) *34*, 1932, p. 196; (2) *36*, 1933, p. 304. Cf. the seventh Abschnitt in J. A. Schouten, Der Ricci-Kalkül, Berlin, 1924. For the alternating group see G. Frobenius, Sitzungsber. Preuss. Akad. 1901, p. 303; concerning the octahedral group which will play a casual rôle in Chapter VII: A. Young, Proc. London Math. Soc. (2) *31*, 1930, p. 273; W. Specht, Math. Zeitschr. *42*, 1937, p. 120; for generalizations in another direction: W. Specht, Schriften Math. Sem. Berlin, *1*, 1932; Math. Zeitschr. *37*, 1933, p. 321.

[4] (130) The relation between the symmetric group and the full linear group was first discovered and applied to an analysis of the representations of the latter by I. Schur in his Dissertation, Berlin, 1901.

[5] (135) About the earlier history of the invariant theoretic expansions see R. Weitzenböck, Invariantentheorie, Groningen, 1923, p. 137.

CHAPTER V

[1] (137) This method is due to R. Brauer: On Algebras which are connected with the Semisimple Continuous Groups, Ann. of Math. *38*, 1937, p. 857.

[2] (141) R. Brauer, l.c.[1], p. 870.

[3] (141) Math. Zeitschr. *35*, 1932, p. 300.

[4] (149) See section 5 of R. Brauer's paper, l.c.[1]

[5] (158) By the infinitesimal method (Chapter VIII, B) E. Cartan constructed all irreducible representations of any simple continuous group and thus in particular of the orthogonal group $O(n)$ in his paper in Bull. Soc. Math. de France *41*, 1913, p. 53. It needed some further considerations supplied by H. Weyl, Nachr. Gött. Ges. Wissensch. 1927, p. 227 to show that the substrata of his representations are our subspaces $P_0(f_1 \cdots f_\nu)$, viz. the

proof of the following statement that lies in the direction of, but is essentially weaker than, the Theorems (5.2.B) and (5.3.A): Consider all polynomials $\Phi(x^1, \cdots, x^\nu)$ depending on ν arbitrary vectors $x^1, \cdots x^\nu$ which vanish whenever the ν^2 relations

$$(*) \qquad (x^\alpha x^\beta) = 0 \qquad (\alpha, \beta = 1, \cdots, \nu)$$

hold. They form an ideal of which the left sides of (*) constitute an ideal basis.

[6] (159) A. H. Clifford, Ann. of Math. *38*, 1937, p. 533.

[7] (164) The descent from the symmetric to the alternating group could be accomplished in similar manner as this transition from the full to the proper orthogonal group by virtue of A. Clifford's theorem. Cf. G. Frobenius, Sitzungsber. Preuss. Akad. 1901, p. 303.

CHAPTER VI

[1] (167) Weyl, Math. Zeitschr. *20*, 1924, p. 140. The group $Sp(n)$ with a rim of arbitrary width μ and its invariants depending on covariant and contravariant vector arguments is treated in R. Wanner, Dissertation, Zürich, 1926.

[2] (174) Cf. R. Brauer, Ann. of Math. *38*, 1937, p. 855; and H. Weyl, Math. Zeitschr. *35*, 1932, p. 300.

CHAPTER VII

[1] (184) For the fact that (2.10) imply $|A| \neq 0$ cf. H. Minkowski, Nachr. Gött. Ges. Wissensch. 1900, p. 90.

[2] (189) In order to prove the first main theorem for invariants (Chapter VIII B, in particular Theorem 8.14.A) the process was first introduced by A. Hurwitz, Nachr. Gött. Ges. Wissensch. 1897, p. 71, and applied to the real orthogonal group.

[3] (189) In three important papers in the Sitzungsber. Preuss. Akad. 1924, pp. 189, 297, 346, entitled "Neue Anwendungen der Integralrechnung auf Probleme der Invariantentheorie," I. Schur first applied the integration process to the representations of compact groups, especially of the real orthogonal group.

[4] (189) Math. Ann. *97*, 1927, p. 737.

[5] (193) About the whole subject and its literature consult: H. Bohr, Fastperiodische Funktionen, Ergebn. Math. *1*, 5. Berlin, 1932.

[6] (193) Ann. of Math. *34*, 1933, p. 147.

[7] (193) Transact. Am. Math. Soc. *36*, 1934, p. 445. Compare also W. Maak, Abh. Math. Sem. Hamburg, *11*, 1935–36, p. 240.

[8] (194) Ann. of Math. *37*, 1936, p. 57.

[9] (194) E. Cartan, Rend. Circ. Mat. Palermo *53*, 1929, p. 217. H. Weyl, Ann. of Math., *35*, 1934, p. 486.

[10] (194) L. Pontrjagin, Ann. of Math. *35*, 1934, p. 361.

[11] (198) In three papers on the Theorie der Darstellung kontinuierlicher halbeinfacher Gruppen durch lineare Transformationen, Math. Zeitschr. *23*, 1925, p. 271; *24*, 1926, pp. 338 and 377 (Appendix, p. 789) the author combined Lie-Cartan's infinitesimal with Hurwitz-Schur's integral approach. The first paper contains the determination of the class density and the characters of the unitary group by the integral method.

[12] (203) The formula, equating the two expressions (5.15) and (6.5), is originally due to G. Jacobi; see Muir, Theory of Determinants I (1906), p. 341. About the later work of Trudi, Naegelsbach and Kotska, ibid. III (1920), p. 135 and IV (1923), p. 145. A recent generalization by Aitken, Proc. Edin. Math. Soc. *1*, 1927, p. 55; *2*, 1930, p. 164.

[13] (203) First given by I. Schur in his Dissertation, Berlin, 1901.

[14] (208) The proof of Theorem (7.6.F), as of the corresponding theorems for the other classical groups, is a simplified version of the procedure I followed in: Acta Math. *48*, p. 255.

[15] (208) Sitzungsber. Preuss. Akad. 1900, p. 516. For other direct algebraic methods see I. Schur's Dissertation, Berlin, 1901, his papers Sitzungsber. Preuss. Akad. 1908, p. 664, and 1927, p. 58.

[16] (215) Am. Jour. of Math. *59*, 1937, p. 437. Cf. also F. D. Murnaghan, Am. Jour. of Math. *59*, 1937, p. 739; *60*, 1938, p. 44, and G. de B. Robinson, Am. Jour. of Math. *60*, 1938, p. 745.

[17] (215) Math. Zeitschr. *23*, 1925, p. 300.

[18] (215) I. Schur, Sitzungsber. Preuss. Akad. 1908, p. 664. Cf. the résumé in W. Specht, Math. Zeitschr. *39*, 1935, p. 696. Related investigations: A. Young, Proc. London Math. Soc. (1), *34*, 1902, p. 361. D. E. Littlewood and A. R. Richardson, Phil. Transact. Roy. Soc. (A), *233*, 1934, p. 99; Quart. Jour. of Math. (Oxford) *5*, 1934, p. 269. D. E. Littlewood, Proc. London Math. Soc. (2), *39*, 1936, p. 150; (2) *40*, 1936, p. 49; (2) *43*, 1937, p. 226.

[19] (220) Cf. the second of my papers on the theory of representations of semisimple groups, Math. Zeitschr. *24*, 1925, p. 328.

[20] (223) In Math. Zeitschr. *24*, 1925, p. 328, I treated the proper orthogonal group. What I added in Acta Math. *48*, p. 255, so as to cover the case of the full orthogonal group should be replaced by the development here given. A more algebraic deduction of the characters is the subject of R. Brauer's Dissertation, "Ueber die Darstellung der Drehungsgruppe durch Gruppen linearer Substitutionen," Berlin, 1925.

[21] (229) It is easy to carry over the combinatorial approach and the "row-wise" generating function to both the symplectic and orthogonal groups. Cf. F. D. Murnaghan, Nat. Ac. of Sciences *24*, 1938, p. 184. The formula for the number of invariants ($f_\alpha = 0$) is here as in the symplectic case, cf. (8.13), an immediate consequence of the first main theorem. I. Schur, in his papers Sitzungsber. Preuss. Akad. 1924, which inaugurated the application of the integration method to group theory, deduced from this equation the formulae (9.7), (9.15) for the volume measure on the orthogonal group which we secured by a direct geometric computation.

[22] (230) The formula for the orthogonal group was first given in R. Brauer's Dissertation, Berlin, 1925. Both of us have been aware for a long time that the same formula holds for any semisimple group; I give here my proof. Cf. Brauer's note, Comptes rendus, *204*, 1937, p. 1784. An explicit rule for the $\times$-multiplication of the two irreducible representations $\langle P(f_1 \cdots f_n)\rangle$, $\langle P(f' \cdots f_n)\rangle$ of the full linear group in: D. E. Littlewood and A. R. Richardson, Phil. Trans. Roy. Soc. (A), *233*, 1934, p. 99. Also F. D. Murnaghan, Am. Jour. of Math. *60*, 1938, p. 761.

[23] (234) Comptes Rendus *201*, 1935, p. 419. E. Cartan had guessed the correct result before: Ann. Soc. Polon. de Math. *8*, 1929, p. 181. See R. Brauer's own detailed account in the mimeographed notes of my lectures On the Structure and Representations of Continuous Groups II, Princeton, 1934-35.

CHAPTER VIII

[1] (239) The following textbooks are in the classic tradition: I. H. Grace and A. Young, The Algebra of Invariants, Cambridge, 1903. Glenn, The Theory of Invariants, Boston, 1915. L. E. Dickson, Algebraic Invariants, New York, 1913. A freer attitude as to the underlying group of transformations is taken in: R. Weitzenböck, Invariantentheorie, Groningen, 1923.

[2] (242) H. Weyl, Rend. Circ. Mat. Palermo *48*, 1924, p. 29.

[3] (250) See e.g. I. H. Grace and A. Young, The Algebra of Invariants, Cambridge, 1903, pp. 89-91, 96-97. The result was first derived by Cayley in his Memoirs on Quantics.

[4] (251) D. Hilbert, Math. Ann. *36*, 1890, pp. 473-534, = Gesammelte Abhandlungen II, Berlin, 1933, No. 16: "Ueber die Theorie der algebraischen Formen," Theorems I and II on pp. 199 and 211. van der Waerden, Moderne Algebra II, Berlin, 1931, pp. 23-24. The finiteness of an ideal basis for every ideal in R is equivalent to E. Noether's "Teilerkettensatz," cf. l.c., pp. 25-27.

[5] (254) The decisive facts are given in Hilbert's paper quoted under[4], including the theory of syzygies into which we did not enter here. A more detailed study of the ring of invariants and its quotient field aiming at a more finitistic construction of the integrity

basis is contained in Hilbert's later paper "Ueber die vollen Invariantensysteme," Math. Ann. *42*, 1893, pp. 313–373, = Gesammelte Abhandlungen II, No. 19, pp. 287–344. For a simpler proof of his "zero theorem" (p. 294) see A. Rabinowitsch, Math. Ann. *102*, 1929, p. 518, and van der Waerden, Moderne Algebra II, p. 11. The "zero manifold" consists of the sets of values $u, v \cdots$ for which all non-constant invariants $J(u, v, \cdots)$ vanish, and its construction as the intersection $J_1 = 0, \cdots, J_h = 0$ by means of a number of invariants $J_1, \cdots, J_h$ whose weights can be limited a priori plays an important rôle. Useful in this connection is a general criterion of finiteness due to E. Noether: Nachr. Gött. Ges. Wissensch. 1926, p. 28.

[6] (255) The idea of adjunction was emphasized by F. Klein, Erlanger program, passim.

[7] (258) This notion is due to O. Schreier, Abh. Math. Sem. Hamburg *4*, 1926, p. 15, and *5*, 1927, p. 233.

[8] (258) See H. Weyl, Die Idee der Riemannschen Fläche, Leipzig, 1913 (and 1923), §9. The idea of the universal covering manifold goes back to H. A. Schwarz and H. Poincaré (H. Poincaré, Bull. Soc. Math. de France *11*, 1883, pp. 113–114). For a genetic construction see P. Koebe, Jour. reine angew. Math. *139*, 1911, pp. 271–276. For the topological study of continuous groups in general see E. Cartan's two pamphlets: La théorie des groupes finis et continus et l'Analysis situs, Mem. des Sciences Math. *42*, Paris, 1930, and La topologie des groupes de Lie, Actual. Scient. *358*, Paris, 1936.

[9] (260) Lie-Engel, Theorie der Transformationsgruppen, 3 vols., Leipzig, 1893. More recent presentations: H. Weyl, Appendix 8 in Mathematische Analyse des Raumproblems, Berlin, 1923; L. P. Eisenhart, Continuous Groups of Transformations, Princeton, 1933; W. Mayer and T. Y. Thomas, Ann. of Math. *36*, 1935, p. 770. For a simplified treatment of the most important parts of E. Cartan's work on infinitesimal groups (cf. Chap. II[8]), see the author's papers in Math. Zeitschr. *23* and *24* (1925–26), and van der Waerden, Math. Zeitschr. *37*, 1933, p. 446. The construction of all (semi-) simple Lie algebras in $K^\dagger$ (or more generally in an algebraically closed field) has been the pivot of these investigations, as far as they deal with the structure rather than with the representations of groups. The same problem in an arbitrary field has recently been successfully attacked by N. Jacobson, Ann. of Math. *36*, 1935, p. 875; *38*, 1937, p. 508; Proc. Nat. Ac. of Sciences *23*, 1937, p. 240, and by W. Landherr, Abh. Math. Sem. Hamburg *11*, 1935, p. 41. Given an associative algebra $\mathfrak{a} = \{a\}$ with an involutorial anti-automorphism J: $a \to a^J$, its J-skew elements a satisfying $a^J = -a$ form a Lie algebra under the multiplication $[ab] = ab - ba$: this procedure of constructing Lie algebras has proved of paramount importance.

[9a] (260) For Lie groups this doubt has been settled by E. Cartan, Actual. Scient. *358*, 1936, p. 19.

[10] (267) I. Schur, Sitzungsber. Preuss. Akad. 1928, p. 96.

[11] (268) H. Weyl, Math. Zeitschr. *24*, 1926, pp. 348–353.

[12] (268) E. Mohr, Dissertation, Göttingen, 1933.

[13] (268) R. Brauer, Sitzungsber. Preuss. Akad., 1929, p. 3.

[14] (268) J. v. Neumann, Math. Zeitschr. *30*, 1929, p. 3. E. Cartan, Mémor. Sc. Math. *42*, 1930, pp. 22–24.

[15] (268) The author's original derivation of the connectivity of the classical groups in Math. Zeitschr. *23*, 1925, p. 291, and *24*, 1925, pp. 337 and 346, is more complicated. For arbitrary semi-simple groups see ibid. *24*, 1925, p. 380; E. Cartan, Annali di Mat. (4) *4*, 1926–27, p. 209, and (4) *5*, 1928, p. 253; Weyl, Mimeographed Notes on the Structure and Representation of Continuous Groups II, Princeton, 1934–1935, pp. 155–185.

[16] (269) E. Cartan, Bull. Soc. Math. de France *41*, 1913, p. 53. P. A. M. Dirac, Proc. Roy. Soc. (A), *117*, 1927, p. 610; *118*, 1928, p. 351. R. Brauer and H. Weyl, Am. Jour. of Math. *57*, 1935, p. 425. A detailed geometric study of the problem is contained in the mimeographed notes On the Geometry of Complex Domains by O. Veblen and J. W. Givens, Princeton, 1935–36.

[17] (270) The algebra was introduced by W. K. Clifford as early as 1878: Am. Jour. of Math. *1*, 1878, p. 350, = Math. Papers, p. 271. An interesting application of this algebra

was made by H. Witt for the study of quadratic forms in arbitrary fields, Jour. reine angew. Math. *176*, 1937, p. 31.

[18] (275) Attempts which miscarried were made by L. Maurer, Bayer. Akad. Wissensch. *29*, 1899, p. 147; Math. Ann. *57*, 1903, p. 265, and R. Weitzenböck, Acta Math. *58*, 1932, p. 231. Weitzenböck's paper contains a correct proof for any individual linear transformation. By an interesting direct algebraic approach, E. Fischer, Jour. reine angew. Math. *140*, 1911, p. 48, settles the question in $K^{\dagger}$ for each linear group which contains the transposed conjugate $\bar{A}^*$ of any of its elements A.

[19] (275) E. Noether, Math. Ann. 77, 1916, p. 89. The same result for finite groups in a field of prime characteristic (dividing the order of the group): E. Noether, Nachr. Gött. Ges. Wissensch. 1926, p. 28. Projective invariants mod p were treated before by L. E. Dickson and his school: On Invariants and the Theory of Numbers, Madison Colloquium, 1913, and various papers during the following years in the Transact. Am. Math. Soc.

[20] (276) E. Cartan, Leçons sur les invariants intégraux, Paris, 1922; Ann. Soc. Polon. de Math. *8*, 1929, p. 181. E. Kähler, Einführung in die Theorie der Systeme von Differentialgleichungen, Leipzig, 1934. J. H. C. Whitehead, Quart. Jour. of Math. (Oxford) *8*, 1937, p. 220.

[21] (277) The coincidence of both definitions was proved by G. de Rham in his Thèse, Paris, 1931 (= Jour. Math. pures et appl. (9), *10*, 1931, p. 165), where he carefully lays the foundation of this theory.

[22] (277) J. W. Alexander, Ann. of Math. *37*, 1936, p. 698. The same idea was presented by A. Kolmogoroff at the Topological Conference in Moscow, Sept. 1935. Furthermore E. Čech, Ann. of Math. *37*, 1936, p. 681.

[23] (279) A direct topological approach: L. Pontrjagin, Comptes Rendus *200*, 1935, p. 1277.

CHAPTER IX

[1] (280) I follow my own method as expounded in Ann. of Math. *37*, 1936, pp. 743–745, and *38*, 1937, pp. 477–483. For the abstract treatment see: van der Waerden, Moderne Algebra II, pp. 172–177, 207–211. Deuring, Algebren, Ergebn. Math. *4*, 1, Berlin, 1935, and the literature cited there. Particularly important: E. Noether, Math. Zeitschr. *37*, 1933, p. 514.

[2] (282) First proved by Th. Skolem, Shr. norske Vid.-Akad., Oslo, 1927.

[3] (287) R. Brauer, Jour. reine angew. Math. *166*, 1932, p. 241; *168*, 1932, p. 44.

[4] (290) Cf. R. Brauer and E. Noether, Sitzungsber. Preuss. Akad. 1927, p. 221. Concerning E. Noether's related "verschränkte Produkte" and R. Brauer's "Faktorensysteme" see: H. Hasse, Transact. Am. Math. Soc. *34*, 1932, p. 171; R. Brauer, Math. Zeitschr. *28*, 1928, p. 677; *31*, 1930, p. 733; also Weyl, Ann. of Math. *37*, 1936, pp. 723–728, and Deuring, l.c.

[5] (290) van der Waerden, Moderne Algebra II, p. 174. J. H. M. Wedderburn, Ann. of Math. *38*, 1937, p. 854.

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For Chapters VII and VIII compare now: D. E. Littlewood, Proc. Camb. Phil. Soc. *38*, 1942, 394–396; ibid. *39*, 1943, 197–199; Phil. Trans. Roy. Soc. London Ser. A, *239*, 1944, 305–365 and 387–417.

About Hodge's theory of harmonic integrals, which is related to the subject of Chap. VIII, §16, cf. H. Weyl, Ann. of Math. *44*, 1945, 1–6. Pontrjagin's method for determining the Betti numbers of compact Lie groups [see Bibliography, Chap. VIII,[23]] is more fully developed in: Rec. Math., New Ser. (Mat. Sbornik) (6) *48*, 1939, 389–422. Related papers: H. Hopf, Ann. of Math. *42*, 1941, 22–52, and H. Samelson, ibid., 1093–1137.

For the construction of all semi-simple Lie algebras compare, besides the papers mentioned in the Bibliography, Chap. VIII,[9]: E. Witt, Abh. Math. Sem. Hans. Univ. *14*, 1941, 289–322.

For Chap. X, Suppl. A and B, cf. H. Weyl, Amer. Jour. of Math. *63*, 1941, 779–784; for Suppl. D cf. N. Jacobson, Theory of Rings, Math. Surveys *2*, 1943, Chapter 5.