

corporate optimization procedures to decrease conservatism, which is not the case in the stability results cited above. Finally, for additional references on PWM feedback control systems (not necessarily dealing with stability), the reader may want to consult [4], [11], [18], [24], [25], [49], [50], [52] - [54], [62], [72], [74] - [76], [80], and the survey paper by Tsafestas [81].

Good references on *variable structure systems* with applications in control are [17] and [82]. The results of Section 10.4 are based on [56].

Applications of *linear systems with state saturation constraints* arise in numerous disciplines, including feedback control systems, artificial neural networks, signal processing, and others. The reader may wish to consult the monograph [59], the special issue of the *International Journal of Robust and Nonlinear Control* [46], the survey paper [9], and the many references cited in these sources to get an idea of the scope of some of the literature dealing with systems with saturation nonlinearities. Subsections 10.5A through 10.5D are based on material in [31] - [34], the results in Subsection 10.5E are from [31], while Subsection 10.5F is based on some of the results contained in [57]. In [34], *necessary and sufficient conditions* for the asymptotic stability of second order (planar) linear systems subjected to saturation nonlinearities are also presented. Unfortunately, the method employed to obtain these results cannot be extended to systems of order higher than 2. This statement is also true for essentially identical results that were obtained in [61] for planar systems. The results in [34] and [61] were discovered concurrently and independently.

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