

# Contents

1	<b>Mathematician's World</b>	1
1.1	Mathematical Structures	2
1.2	Everything Is a Set	25
1.3	Antinomies of Set Theory	36
1.4	The Axiomatic Method	43
1.5	The Necessity of Using Abstract Concepts	54
	Main Points of the Chapter	64
2	<b>Language, Logic and Computations</b>	65
2.1	The Language of Mathematics	66
2.2	Truth and Models	80
2.3	Proofs	92
2.4	Programs and Computations	123
2.5	The Lambda Calculus	146
	Main Points of the Chapter	155
3	<b>Set Theory</b>	157
3.1	The Axioms of Set Theory	159
3.2	The Arithmetic of Infinity	176
3.3	What Is the Largest Number?	196
3.4	Controversial Axioms	215
3.5	Alternative Set-Theoretical Foundations	231
	Main Points of the Chapter	253
4	<b>Proofs of Impossibility</b>	255
4.1	Impossibility Proofs in Geometry and Algebra	256
4.2	The Incompleteness Theorems	272
4.3	Algorithmically Unsolvable Problems	300
4.4	Concrete Independence	319
4.5	The Independent Sentences of Set Theory	340
	Main Points of the Chapter	364

<b>5</b>	<b>The Complexity of Computations</b>	365
5.1	What Is Complexity?	366
5.2	Randomness, Interaction and Cryptography	410
5.3	Parallel Computations	437
5.4	Quantum Computations	448
5.5	Descriptional Complexity	479
	Main Points of the Chapter	493
<b>6</b>	<b>Proof Complexity</b>	495
6.1	Proof Theory	496
6.2	Theories and Complexity Classes	523
6.3	Propositional Proofs	540
6.4	Feasible Incompleteness	562
	Main Points of the Chapter	580
<b>7</b>	<b>Consistency, Truth and Existence</b>	583
7.1	Consistency and Existence	584
7.2	The Attributes of Reality	609
7.3	Finitism and Physical Reality	646
	Main Points of the Chapter	664
	<b>Bibliographical Remarks</b>	667
	<b>References</b>	671
	<b>Name Index</b>	683
	<b>Subject Index</b>	687
	<b>Symbols and Abbreviations</b>	695