

---

# Contents

<b>1</b>	<b>Introduction</b> .....	<b>1</b>
<b>2</b>	<b>Elementary Fundamentals</b> .....	<b>11</b>
2.1	Introduction .....	11
2.2	Fundamentals of Mathematics .....	13
2.2.1	Linear Algebra .....	13
2.2.2	Combinatorics, Counting, and Graph Theory .....	30
2.2.3	Boolean Functions and Formulae .....	45
2.2.4	Algebra and Number Theory .....	54
2.2.5	Probability Theory .....	80
2.3	Fundamentals of Algorithmics .....	93
2.3.1	Alphabets, Words, and Languages .....	93
2.3.2	Algorithmic Problems .....	97
2.3.3	Complexity Theory .....	114
2.3.4	Algorithm Design Techniques .....	134
<b>3</b>	<b>Deterministic Approaches</b> .....	<b>149</b>
3.1	Introduction .....	149
3.2	Pseudo-Polynomial-Time Algorithms .....	152
3.2.1	Basic Concept .....	152
3.2.2	Dynamic Programming and Knapsack Problem .....	154
3.2.3	Maximum Flow Problem and Ford-Fulkerson Method .....	157
3.2.4	Limits of Applicability .....	167
3.3	Parameterized Complexity .....	169
3.3.1	Basic Concept .....	169
3.3.2	Applicability of Parameterized Complexity .....	171
3.3.3	Discussion .....	174
3.4	Branch-and-Bound .....	175
3.4.1	Basic Concept .....	175
3.4.2	Applications for MAX-SAT and TSP .....	177
3.4.3	Discussion .....	183

3.5	Lowering Worst Case Complexity of Exponential Algorithms . . . . .	184
3.5.1	Basic Concept . . . . .	184
3.5.2	Solving 3SAT in Less than $2^n$ Complexity . . . . .	185
3.6	Local Search . . . . .	189
3.6.1	Introduction and Basic Concept . . . . .	189
3.6.2	Examples of Neighborhoods and Kernighan-Lin's Variable-Depth Search . . . . .	193
3.6.3	Tradeoffs Between Solution Quality and Complexity . . . . .	198
3.7	Relaxation to Linear Programming . . . . .	209
3.7.1	Basic Concept . . . . .	209
3.7.2	Expressing Problems as Linear Programming Problems . . . . .	211
3.7.3	The Simplex Algorithm . . . . .	218
3.7.4	Rounding, LP-Duality and Primal-Dual Method . . . . .	227
3.8	Bibliographical Remarks . . . . .	243
<b>4</b>	<b>Approximation Algorithms . . . . .</b>	<b>247</b>
4.1	Introduction . . . . .	247
4.2	Fundamentals . . . . .	248
4.2.1	Concept of Approximation Algorithms . . . . .	248
4.2.2	Classification of Optimization Problems . . . . .	253
4.2.3	Stability of Approximation . . . . .	253
4.2.4	Dual Approximation Algorithms . . . . .	258
4.3	Algorithm Design . . . . .	260
4.3.1	Introduction . . . . .	260
4.3.2	Cover Problems, Greedy Method, and Relaxation to Linear Programming . . . . .	261
4.3.3	Maximum Cut Problem and Local Search . . . . .	269
4.3.4	Knapsack Problem and PTAS . . . . .	272
4.3.5	Traveling Salesperson Problem and Stability of Approximation . . . . .	282
4.3.6	Bin-Packing, Scheduling, and Dual Approximation Algorithms . . . . .	308
4.4	Inapproximability . . . . .	316
4.4.1	Introduction . . . . .	316
4.4.2	Reduction to NP-Hard Problems . . . . .	317
4.4.3	Approximation-Preserving Reductions . . . . .	319
4.4.4	Probabilistic Proof Checking and Inapproximability . . . . .	329
4.5	Bibliographical Remarks . . . . .	337
<b>5</b>	<b>Randomized Algorithms . . . . .</b>	<b>341</b>
5.1	Introduction . . . . .	341
5.2	Classification of Randomized Algorithms and Design Paradigms . . . . .	343
5.2.1	Fundamentals . . . . .	343
5.2.2	Classification of Randomized Algorithms . . . . .	345
5.2.3	Paradigms of Design of Randomized Algorithms . . . . .	359

5.3	Design of Randomized Algorithms . . . . .	363
5.3.1	Introduction . . . . .	363
5.3.2	Quadratic Residues, Random Sampling, and Las Vegas . . . . .	364
5.3.3	Primality Testing, Abundance of Witnesses, and One-Sided-Error Monte Carlo . . . . .	369
5.3.4	Equivalence Tests, Fingerprinting, and Monte Carlo . . . . .	385
5.3.5	Randomized Optimization Algorithms for MIN-CUT . . . . .	392
5.3.6	MAX-SAT and Random Rounding . . . . .	400
5.3.7	3SAT and Randomized Multistart Local Search . . . . .	406
5.4	Derandomization . . . . .	411
5.4.1	Fundamental Ideas . . . . .	411
5.4.2	Derandomization by the Reduction of the Probability Space Size . . . . .	413
5.4.3	Probability Space Reduction and MAX-E $k$ SAT . . . . .	418
5.4.4	Derandomization by the Method of Conditional Probabilities . . . . .	420
5.4.5	Conditional Probabilities and Satisfiability . . . . .	422
5.5	Bibliographical Remarks . . . . .	426
<b>6</b>	<b>Heuristics . . . . .</b>	<b>431</b>
6.1	Introduction . . . . .	431
6.2	Simulated Annealing . . . . .	433
6.2.1	Basic Concept . . . . .	433
6.2.2	Theory and Experience . . . . .	437
6.2.3	Randomized Tabu Search . . . . .	441
6.3	Genetic Algorithms . . . . .	444
6.3.1	Basic Concept . . . . .	444
6.3.2	Adjustment of Free Parameters . . . . .	452
6.4	Bibliographical Remarks . . . . .	457
<b>7</b>	<b>A Guide to Solving Hard Problems . . . . .</b>	<b>461</b>
7.1	Introduction . . . . .	461
7.2	Taking over an Algorithmic Task or a Few Words about Money . . . . .	462
7.3	Combining Different Concepts and Techniques . . . . .	463
7.4	Comparing Different Approaches . . . . .	466
7.5	Speedup by Parallelization . . . . .	468
7.6	New Technologies . . . . .	477
7.6.1	Introduction . . . . .	477
7.6.2	DNA Computing . . . . .	479
7.6.3	Quantum Computing . . . . .	486
7.7	Glossary of Basic Terms . . . . .	492
	<b>References . . . . .</b>	<b>503</b>
	<b>Index . . . . .</b>	<b>525</b>