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## 1.1 Conventional optics and nonlinear optics

The principles of the light propagation in a medium are well known. In 1802, the first law of diffraction (the Huygens–Fresnel law) was discovered by Fresnel in 1801–1802. Fresnel is a French physicist and mathematician, who was particularly distinguished by his contributions to optics and to acoustics. The comprehensive studies of these optical effects and their applications are the foundation of conventional optics and nonlinear optics.

In conventional optics, which was established originally in the 17th century, some basic mathematical equations in differential equations are used to describe the light propagation. The following are several typical examples starting with a medium of conventional optics.

• First, in order to describe the propagation of light in a medium, we should consider an infinitesimal physical system, the light intensity, phase, polarization of the medium in the region of conventional optics. The electric polarization vector  $P$  can simply be assumed to be linear proportional to the electric field strength  $E$  of an applied electromagnetic wave, i.e.,

$$P = \epsilon_0 \chi E, \quad (1.1.1)$$

where  $\epsilon_0$  is the permittivity of vacuum and  $\chi$  is the susceptibility of a given medium. With this linear assumption, Maxwell's equations lead to a set of linear differential equations governing the wave propagation in the frequency domain of the field  $E$ . As a result, the wavefront nature of the medium is completely neglected, which is independent of the light frequency, and there will be no coupling between the light waves of different frequencies as a transparent medium is transparent to light regardless of the wave frequencies and frequencies.

• Second, to understand the wave propagation of a light wave propagating through an absorptive medium can be described as