

Contents

1 Basic Concepts in Banach Spaces	1
1.1 Basic Definitions	1
1.2 Hölder and Minkowski Inequalities, Classical Spaces $C[0, 1]$, ℓ_p , c_0 , $L_p[0, 1]$	3
1.3 Operators, Quotients, Finite-Dimensional Spaces	13
1.4 Hilbert Spaces	24
1.5 Remarks and Open Problems	29
Exercises for Chapter 1	31
2 Hahn–Banach and Banach Open Mapping Theorems	53
2.1 Hahn–Banach Extension and Separation Theorems	54
2.2 Duals of Classical Spaces	60
2.3 Banach Open Mapping Theorem, Closed Graph Theorem, Dual Operators	65
2.4 Remarks and Open Problems	68
Exercises for Chapter 2	68
3 Weak Topologies and Banach Spaces	83
3.1 Dual Pairs, Weak Topologies	83
3.2 Topological Vector Spaces	86
3.3 Locally Convex Spaces	94
3.4 Polarity	98
3.5 Topologies Compatible with a Dual Pair	100
3.6 Topologies of Subspaces and Quotients	103
3.7 Weak Compactness	104
3.8 Extreme Points, Krein–Milman Theorem	109
3.9 Representation and Compactness	112
3.10 The Space of Distributions	115
3.11 Banach Spaces	119
3.11.1 Banach–Steinhaus Theorem	119
3.11.2 Banach–Dieudonné Theorem	122

3.11.3	The Bidual Space	125
3.11.4	The Completion of a Normed Space	126
3.11.5	Separability and Metrizable	127
3.11.6	Weak Compactness	129
3.11.7	Reflexivity	129
3.11.8	Boundaries	131
3.12	Remarks and Open Problems	141
	Exercises for Chapter 3	142
4	Schauder Bases	179
4.1	Projections and Complementability, Auerbach Bases	179
4.2	Basics on Schauder Bases	182
4.3	Shrinking and Boundedly Complete Bases, Perturbation	187
4.4	Block Bases, Bessaga–Pelczyński Selection Principle	194
4.5	Unconditional Bases	200
4.6	Bases in Classical Spaces	205
4.7	Subspaces of L_p Spaces	213
4.8	Markushevich Bases	216
4.9	Remarks and Open Problems	218
	Exercises for Chapter 4	220
5	Structure of Banach Spaces	237
5.1	Extension of Operators and Lifting	237
5.2	Weak Injectivity	250
5.2.1	Schur Property	252
5.3	Rosenthal's ℓ_1 Theorem	253
5.4	Remarks and Open Problems	264
	Exercises for Chapter 5	267
6	Finite-Dimensional Spaces	291
6.1	Finite Representability	291
6.2	Spreading Models	294
6.3	Complemented Subspaces in Spaces with an Unconditional Schauder Basis	298
6.4	The Complemented-Subspace Result	309
6.5	The John Ellipsoid	312
6.6	Kadec–Snobar Theorem	320
6.7	Grothendieck's Inequality	323
6.8	Remarks	325
	Exercises for Chapter 6	326
7	Optimization	331
7.1	Introduction	331
7.2	Subdifferentials: Šmulyan's Lemma	336

7.3	Ekeland Principle and Bishop–Phelps Theorem	351
7.4	Smooth Variational Principle	355
7.5	Norm-Attaining Operators	359
7.6	Michael’s Selection Theorem	361
7.7	Remarks and Open Problems	364
	Exercises for Chapter 7	365
8	C^1-Smoothness in Separable Spaces	383
8.1	Smoothness and Renormings in Separable Spaces	383
8.2	Equivalence of Separable Asplund Spaces	385
8.3	Applications in Convexity	394
8.4	Smooth Approximation	402
8.5	Ranges of Smooth Maps	405
8.6	Remarks and Open Problems	408
	Exercises for Chapter 8	410
9	Superreflexive Spaces	429
9.1	Uniform Convexity and Uniform Smoothness, ℓ_p and L_p Spaces	429
9.2	Finite Representability, Superreflexivity	435
9.3	Applications	449
9.4	Remarks	453
	Exercises for Chapter 9	453
10	Higher Order Smoothness	465
10.1	Introduction	465
10.2	Smoothness in ℓ_p	466
10.3	Countable James Boundary	468
10.4	Remarks and Open Problems	474
	Exercises for Chapter 10	475
11	Dentability and Differentiability	479
11.1	Dentability in X	479
11.2	Dentability in X^*	486
11.3	The Radon–Nikodým Property	490
11.4	Extension of Rademacher’s Theorem	504
11.5	Remarks and Open Problems	510
	Exercises for Chapter 11	511
12	Basics in Nonlinear Geometric Analysis	521
12.1	Contractions and Nonexpansive Mappings	521
12.2	Brouwer and Schauder Theorems	526

12.3	The Homeomorphisms of Convex Compact Sets: Keller's Theorem	533
12.3.1	Introduction	533
12.3.2	Elliptically Convex Sets	535
12.3.3	The Space T	537
12.3.4	Compact Elliptically Convex Subsets of ℓ_2	538
12.3.5	Keller Theorem	541
12.3.6	Applications to Fixed Points	541
12.4	Homeomorphisms: Kadec's Theorem	542
12.5	Lipschitz Homeomorphisms	545
12.6	Remarks and Open Problems	559
	Exercises for Chapter 12	561
13	Weakly Compactly Generated Spaces	575
13.1	Introduction	575
13.2	Projectional Resolutions of the Identity	577
13.3	Consequences of the Existence of a Projectional Resolution	581
13.4	Renormings of Weakly Compactly Generated Banach Spaces	586
13.5	Weakly Compact Operators	591
13.6	Absolutely Summing Operators	592
13.7	The Dunford–Pettis Property	596
13.8	Applications	598
13.9	Remarks and Open Problems	602
	Exercises for Chapter 13	603
14	Topics in Weak Topologies on Banach Spaces	617
14.1	Eberlein Compact Spaces	617
14.2	Uniform Eberlein Compact Spaces	622
14.3	Scattered Compact Spaces	625
14.4	Weakly Lindelöf Spaces, Property C	629
14.5	Weak* Topology of the Dual Unit Ball	634
14.6	Remarks and Open Problems	642
	Exercises for Chapter 14	643
15	Compact Operators on Banach Spaces	657
15.1	Compact Operators	657
15.2	Spectral Theory	661
15.3	Self-Adjoint Operators	668
15.4	Remarks and Open Problems	678
	Exercises for Chapter 15	678
16	Tensor Products	687
16.1	Tensor Products and Their Topologies	687
16.2	Duality of Injective Tensor Products	696

16.3	Approximation Property and Duality of Spaces of Operators	700
16.4	The Trace	708
16.5	Banach Spaces Without the Approximation Property	711
16.6	The Bounded Approximation Property	717
16.7	Schauder Bases in Tensor Products	721
16.8	Remarks and Open Problems	726
	Exercises for Chapter 16	727
17	Appendix	733
17.1	Basics in Topology	733
17.2	Nets and Filters	735
17.3	Nets and Filters in Topological Spaces	736
17.4	Ultraproducts	737
17.5	The Order Topology on the Ordinals	737
17.6	Continuity of Set-Valued Mappings	738
17.7	The Cantor Space	739
17.8	Baire's Great Theorem	741
17.9	Polish Spaces	741
17.10	Uniform Spaces	741
17.11	Nets and Filters in Uniform Spaces	742
17.12	Partitions of Unity	743
17.13	Measure and Integral	744
	17.13.1 Measure	744
	17.13.2 Integral	745
17.14	Continued Fractions and the Representation of the Irrational Numbers	746
	References	751
	Symbol Index	777
	Subject Index	781
	Author Index	807