

Contents

Preface	x	xi
Preface to the English edition	xi	
A letter from the doctoral student Jiří Pecha prior to publication of the lectures	xii	
A letter from the authors to the doctoral student Jiří Pecha	xiii	
Basic concepts and notations	xv	
Acknowledgements		xix
Lecture 1 Bayesian statistical decision making		1
1.1 Introduction to the analysis of the Bayesian task	1	
1.2 Formulation of the Bayesian task	1	
1.3 Two properties of Bayesian strategies	3	
1.4 Two particular cases of the Bayesian task	7	
1.4.1 Probability of the wrong estimate of the state	7	
1.4.2 Bayesian strategy with possible rejection	9	
1.5 Discussion	11	
1.6 Bibliographical notes	22	
Lecture 2 Non-Bayesian statistical decision making		25
2.1 Severe restrictions of the Bayesian approach	25	
2.1.1 Penalty function	25	
2.1.2 <i>A priori</i> probability of situations	26	
2.1.3 Conditional probabilities of observations	27	
2.2 Formulation of the known and new non-Bayesian tasks	28	
2.2.1 Neyman–Pearson task	28	
2.2.2 Generalised task with two dangerous states	31	
2.2.3 Minimax task	31	
2.2.4 Wald task	32	
2.2.5 Statistical decision tasks with non-random interventions	33	

2.3	The pair of dual linear programming tasks, properties and solutions	35
2.4	The solution of non-Bayesian tasks using duality theorems	40
2.4.1	Solution of the Neyman–Pearson task	41
2.4.2	Solution of generalised Neyman–Pearson task with two dangerous states	44
2.4.3	Solution of the minimax task	46
2.4.4	Solution of Wald task for the two states case	48
2.4.5	Solution of Wald task in the case of more states	50
2.4.6	Testing of complex random hypotheses	52
2.4.7	Testing of complex non-random hypotheses	53
2.5	Comments on non-Bayesian tasks	53
2.6	Discussion	54
2.7	Bibliographical notes	71
Lecture 3	Two statistical models of the recognised object	73
3.1	Conditional independence of features	73
3.2	Gaussian probability distribution	75
3.3	Discussion	77
3.4	Bibliographical notes	99
Lecture 4	Learning in pattern recognition	101
4.1	Myths about learning in pattern recognition	101
4.2	Three formulations of learning tasks in pattern recognition	102
4.2.1	Learning according to the maximal likelihood	104
4.2.2	Learning according to a non-random training set	105
4.2.3	Learning by minimisation of empirical risk	106
4.3	Basic concepts and questions of the statistical theory of learning	108
4.3.1	Informal description of learning in pattern recognition . .	108
4.3.2	Foundations of the statistical learning theory according to Chervonenkis and Vapnik	113
4.4	Critical view of the statistical learning theory	120
4.5	Outlines of deterministic learning	122
4.6	Discussion	127
4.7	Bibliographical notes	136
Lecture 5	Linear discriminant function	137
5.1	Introductory notes on linear decomposition	137
5.2	Guide through the topic of the lecture	138
5.3	Anderson tasks	141
5.3.1	Equivalent formulation of generalised Anderson task	141
5.3.2	Informal analysis of generalised Anderson task	142
5.3.3	Definition of auxiliary concepts for Anderson tasks	145
5.3.4	Solution of Anderson original task	147
5.3.5	Formal analysis of generalised Anderson task	150

5.3.6	Outline of a procedure for solving generalised Anderson task	157
5.4	Linear separation of finite sets of points	159
5.4.1	Formulation of tasks and their analysis	159
5.4.2	Algorithms for linear separation of finite sets of points . .	163
5.4.3	Algorithm for ϵ -optimal separation of finite sets of points by means of the hyperplane	167
5.4.4	Construction of Fisher classifiers by modifying Kozinec and perceptron algorithms	169
5.4.5	Further modification of Kozinec algorithms	171
5.5	Solution of the generalised Anderson task	175
5.5.1	ϵ -solution of Anderson task	175
5.5.2	Linear separation of infinite sets of points	179
5.6	Discussion	182
5.7	Link to a toolbox	213
5.8	Bibliographical notes	213
Lecture 6	Unsupervised learning	215
6.1	Introductory comments on the specific structure of the lecture .	215
6.2	Preliminary and informal definition of unsupervised learning . .	217
6.3	Unsupervised learning in a perceptron	219
6.4	Empirical Bayesian approach after H. Robbins	226
6.5	Quadratic clustering and formulation of a general clustering task	232
6.6	Unsupervised learning algorithms and their analysis	238
6.6.1	Formulation of a recognition task	238
6.6.2	Formulation of a learning task	238
6.6.3	Formulation of an unsupervised learning task	240
6.6.4	Unsupervised learning algorithm	241
6.6.5	Analysis of the unsupervised learning algorithm	242
6.6.6	Algorithm solving Robbins task and its analysis	251
6.7	Discussion	253
6.8	Link to a toolbox	273
6.9	Bibliographical notes	274
Lecture 7	Mutual relationship of statistical and structural recognition	275
7.1	Statistical recognition and its application areas	275
7.2	Why is structural recognition necessary for image recognition? .	277
7.2.1	Set of observations	277
7.2.2	Set of hidden parameter values for an image	280
7.2.3	The role of learning and unsupervised learning in image recognition	281
7.3	Main concepts necessary for structural analysis	284
7.4	Discussion	288
7.5	Bibliographical notes	305

Lecture 8	Recognition of Markovian sequences	307
8.1	Introductory notes on sequences	307
8.2	Markovian statistical model of a recognised object	308
8.3	Recognition of the stochastic automaton	312
	8.3.1 Recognition of the stochastic automaton; problem formulation	312
	8.3.2 Algorithm for a stochastic automaton recognition	313
	8.3.3 Matrix representation of the calculation procedure	314
	8.3.4 Statistical interpretation of matrix multiplication	316
	8.3.5 Recognition of the Markovian object from incomplete data	318
8.4	The most probable sequence of hidden parameters	321
	8.4.1 Difference between recognition of an object as a whole and recognition of parts that form the object	321
	8.4.2 Formulation of a task seeking the most probable sequence of states	321
	8.4.3 Representation of a task as seeking the shortest path in a graph	321
	8.4.4 Seeking the shortest path in a graph describing the task	323
	8.4.5 On the necessity of formal task analysis	326
	8.4.6 Generalised matrix multiplications	327
	8.4.7 Seeking the most probable subsequence of states	330
8.5	Seeking sequences composed of the most probable hidden parameters	333
8.6	Markovian objects with acyclic structure	338
	8.6.1 Statistical model of an object	338
	8.6.2 Calculating the probability of an observation	339
	8.6.3 The most probable ensemble of hidden parameters	343
8.7	Formulation of supervised and unsupervised learning tasks	344
	8.7.1 The maximum likelihood estimation of a model during learning	345
	8.7.2 Minimax estimate of the model	345
	8.7.3 Tuning of the recognition algorithm	346
	8.7.4 Task of unsupervised learning	347
8.8	Maximum likelihood estimate of the model	347
8.9	Minimax estimate of a statistical model	353
	8.9.1 Formulation of an algorithm and its properties	353
	8.9.2 Analysis of a minimax estimate	356
	8.9.3 Proof of the minimax estimate algorithm of a Markovian mode	366
8.10	Tuning the algorithm that recognises sequences	366
8.11	The maximum likelihood estimate of statistical model	368
8.12	Discussion	372
8.13	Link to a toolbox	395
8.14	Bibliographical notes	395

Lecture 9	Regular languages and corresponding pattern recognition tasks	397
9.1	Regular languages	397
9.2	Other ways to express regular languages	399
9.2.1	Regular languages and automata	399
9.2.2	Regular languages and grammars	400
9.2.3	Regular languages and regular expressions	401
9.2.4	Example of a regular language expressed in different ways	402
9.3	Regular languages respecting faults; best and exact matching . .	404
9.3.1	Fuzzy automata and languages	405
9.3.2	Penalised automata and corresponding languages	406
9.3.3	Simple best matching problem	407
9.4	Partial conclusion after one part of the lecture	409
9.5	Levenstein approximation of a sentence	410
9.5.1	Preliminary formulation of the task	410
9.5.2	Levenstein dissimilarity	411
9.5.3	Known algorithm calculating Levenstein dissimilarity . . .	412
9.5.4	Modified definition of Levenstein dissimilarity and its properties	414
9.5.5	Formulation of the problem and comments to it	417
9.5.6	Formulation of main results and comments to them	418
9.5.7	Generalised convolutions and their properties	420
9.5.8	Formulation of a task and main results in convolution form	427
9.5.9	Proof of the main result of this lecture	429
9.5.10	Nonconvolution interpretation of the main result	440
9.6	Discussion	443
9.7	Link to a toolbox	477
9.8	Bibliographical notes	477
Lecture 10	Context-free languages, their 2-D generalisation, related tasks	479
10.1	Introductory notes	479
10.2	Informal explanation of two-dimensional grammars and languages	480
10.3	Two-dimensional context-free grammars and languages	484
10.4	Exact matching problem. Generalised algorithm of C-Y-K	486
10.5	General structural construction	489
10.5.1	Structural construction defining observed sets	490
10.5.2	Basic problem in structural recognition of images	493
10.5.3	Computational procedure for solving the basic problem . .	494
10.6	Discussion	498
10.7	Bibliographical notes	505
	Bibliography	507
	Index	514