# Quantum Electrodynamics

The title of this Second Edition has been changed from Relativistic Quantum Theory, because of the omission of the chapters on weak interactions and topics in the theory of strong interactions. Several significant additions have been made, including the operator method of calculating the bremsstrahlung cross-section, the calculation of the probabilities of photon-induced pair production and photon decay in a magnetic field, the asymptotic from of the scattering amplitudes at high energies, inelastic scattering of electrons by hadrons, and the transformation of electron-positron pairs into hadrons.

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