Contents

Preface *page* vii

1 Concepts and examples 1
- 1.1 Ordered sets 1
- 1.2 Examples of uses 10
- 1.3 Ordered subsets and extensions 16
- 1.4 Particular elements and subsets 22
- 1.5 Constructing ordered sets from given ones 27
- 1.6 Further topics and references 32
- 1.7 Exercises 38

2 Particular classes of ordered sets 42
- 2.1 Ranked, semimodular, and bipartite ordered sets 43
- 2.2 Ordered sets with forbidden configurations 48
- 2.3 Semilattices and lattices 50
- 2.4 Linearly ordered sets and tournaments 56
- 2.5 Further topics and references 60
- 2.6 Exercises 64

3 Morphisms of ordered sets 67
- 3.1 Isotone and antitone maps: exponentiation 68
- 3.2 Join- and meet-generating sets 70
- 3.3 Closure and dual closure operators 76
- 3.4 Residuated, residual, and Galois maps 80
- 3.5 The Galois connection associated with a binary relation 86
- 3.6 Further topics and references 97
- 3.7 Exercises 101

4 Chains and antichains 107
- 4.1 Dilworth's decomposition theorem 108
- 4.2 Matchings and transversals in a bipartite ordered set 112
- 4.3 The Sperner property 116
- 4.4 Direct products of chains 119

	4.5	Further topics and references	124
	4.6	Exercises	127
5	**Ordered sets and distributive lattices**		130
	5.1	Distributive lattices	131
	5.2	The distributive lattice associated with an ordered set	134
	5.3	Representations of a distributive lattice	138
	5.4	Dualities: preorders–topologies, orders–distributive lattices	144
	5.5	Duality between orders and spindles of linear orders	150
	5.6	Further topics and references	155
	5.7	Exercises	158
6	**Order codings and dimensions**		163
	6.1	Boolean codings and Boolean dimension of an ordered set	164
	6.2	Dimension of an ordered set	169
	6.3	2-dimensional ordered sets	177
	6.4	k-dimension of an ordered set	181
	6.5	Further topics and references	184
	6.6	Exercises	189
7	**Some uses**		192
	7.1	Models of preferences	192
	7.2	Preference aggregation: Arrowian theorems for orders	205
	7.3	The roles of orders in cluster analysis	215
	7.4	Implicational systems, Moore families and Galois data analysis	230
	7.5	Orders in scheduling	243
	7.6	Further topics and references	251
	7.7	Exercises	260
A	**About algorithmic complexity**		270
	A.1	Computational complexity theory	271
	A.2	Complexity results	276
B	**The 58 types of connected ordered sets of size at most 5**		286
C	**The numbers of ordered sets and of types of ordered sets**		288
D	**Documentation marks**		290
	D.1	Internet and the inescapable Google	290
	D.2	Books	291
	D.3	Journals	293
	D.4	Reviews and proceedings	293
	D.5	Software	294
References			296
List of symbols			325
Index			329