Theoretical and Mathematical Physics

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Basic Concepts of String Theory

The purpose of this book is to thoroughly prepare the reader for research in string theory. It is intended as a textbook in the sense that, starting from the basics, the material is presented in a pedagogical and self-contained fashion. The emphasis is on the world-sheet perspective of closed strings and of open strings ending on D-branes, where two-dimensional conformal field theory is the main tool. Compactifications of string theory, with and without fluxes, and string dualities are also discussed from the space-time point of view, i.e. in geometric language. End-of-chapter references have been added to guide the reader intending to pursue further studies or to start research in the topics covered by this book.

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