

# TEXTBOOKS in MATHEMATICS



**Applied Differential Equations: The Primary Course** presents a contemporary treatment of ordinary differential equations, including their applications in engineering and the sciences. The text enables students majoring in a range of fields to obtain a solid foundation. Developed as a primary text for the author's two-semester course offered in an applied mathematics department, the text offers a true alternative to texts originally published for previous generations.

This interesting new approach contains practical techniques for solving the equations as well as corresponding codes for numerical solvers. Many examples and exercises help students master effective solution techniques, including reliable numerical approximations. The author covers traditional material, along with novel approaches to mathematical modeling that harness the capabilities of numerical algorithms and popular computer software packages.

## Features

- Covers basic and advanced topics in differential equations at a level suitable for an undergraduate course
- Presents various methods for analyzing, solving, and visualizing ODEs
- Gives an introduction to several computer software packages, including Maple™, Mathematica®, MATLAB®
- Numerous examples show students how to model real-world problems
- Provides the basic formulas and techniques, making it easy for students to understand the derivations
- Contains many exercises throughout each chapter and review questions

Connecting calculus, modeling, and advanced topics, this text prepares students for more rigorous study of differential equations and their applications. Students will learn how to formulate a mathematical model, solve differential equations analytically and numerically, analyze them qualitatively, and interpret the results.

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**CRC Press**  
Taylor & Francis Group  
an informa business  
www.crcpress.com

6000 Broken Sound Parkway, NW  
Suite 300, Boca Raton, FL 33487  
711 Third Avenue  
New York, NY 10017  
2 Park Square, Milton Park  
Abingdon, Oxon OX14 4RN, UK

K12330

ISBN: 978-1-4398-5104-3

90000



9 781439 851043

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