

Contents

Chapter I. The Direct Methods in the Calculus of Variations	1
1. Lower Semi-continuity	2
Degenerate Elliptic Equations, 4 — Minimal Partitioning Hypersurfaces, 6 — Minimal Hypersurfaces in Riemannian Manifolds, 7 — A General Lower Semi-continuity Result, 8	
2. Constraints	13
Semilinear Elliptic Boundary Value Problems, 14 — Perron's Method in a Variational Guise, 16 — The Classical Plateau Problem, 19	
3. Compensated Compactness	25
Applications in Elasticity, 29 — Convergence Results for Nonlinear Elliptic Equations, 32 — Hardy Space Methods, 35	
4. The Concentration-Compactness Principle	36
Existence of Extremal Functions for Sobolev Embeddings, 42	
5. Ekeland's Variational Principle	51
Existence of Minimizers for Quasi-convex Functionals, 54	
6. Duality	58
Hamiltonian Systems, 60 — Periodic Solutions of Nonlinear Wave Equations, 65	
7. Minimization Problems Depending on Parameters	69
Harmonic Maps with Singularities, 71	
Chapter II. Minimax Methods	74
1. The Finite Dimensional Case	74
2. The Palais-Smale Condition	77
3. A General Deformation Lemma	81
Pseudo-gradient Flows on Banach Spaces, 81 — Pseudo-gradient Flows on Manifolds, 85	
4. The Minimax Principle	87
Closed Geodesics on Spheres, 89	

5. Index Theory	94
Krasnoselskii Genus, 94 — Minimax Principles for Even Functionals, 96 — Applications to Semilinear Elliptic Problems, 98 — General Index Theories, 99 — Ljusternik-Schnirelman Category, 100 — A Geometrical S^1 -Index, 101 — Multiple Periodic Orbits of Hamiltonian Systems, 103	
6. The Mountain Pass Lemma and its Variants	108
Applications to Semilinear Elliptic Boundary Value Problems, 110 — The Symmetric Mountain Pass Lemma, 112 — Application to Semilinear Equations with Symmetry, 116	
7. Perturbation Theory	118
Applications to Semilinear Elliptic Equations, 120	
8. Linking	125
Applications to Semilinear Elliptic Equations, 128 — Applications to Hamiltonian Systems, 130	
9. Parameter Dependence	137
10. Critical Points of Mountain Pass Type	143
Multiple Solutions of Coercive Elliptic Problems, 147	
11. Non-differentiable Functionals	150
12. Ljusternik-Schnirelman Theory on Convex Sets	162
Applications to Semilinear Elliptic Boundary Value Problems, 166	
Chapter III. Limit Cases of the Palais-Smale Condition	169
1. Pohožaev's Non-existence Result	170
2. The Brezis-Nirenberg Result	173
Constrained Minimization, 174 — The Unconstrained Case: Local Compactness, 175 — Multiple Solutions, 180	
3. The Effect of Topology	183
A Global Compactness Result, 184 — Positive Solutions on Annular-Shaped Regions, 190	
4. The Yamabe Problem	194
The Variational Approach, 195 — The Locally Conformally Flat Case, 197 — The Yamabe Flow, 198 — The Proof of Theorem 4.9 (following Ye [1]), 200 — Convergence of the Yamabe Flow in the General Case, 204 — The Compact Case $u_\infty > 0$, 211 — Bubbling: The Case $u_\infty \equiv 0$, 216	

5. The Dirichlet Problem for the Equation of Constant Mean Curvature	220
Small Solutions, 221 — The Volume Functional, 223 — Wente's Uniqueness Result, 225 — Local Compactness, 226 — Large Solutions, 229	
6. Harmonic Maps of Riemannian Surfaces	231
The Euler-Lagrange Equations for Harmonic Maps, 232 — Bochner identity, 234 — The Homotopy Problem and its Functional Analytic Setting, 234 — Existence and Non-existence Results, 237 — The Heat Flow for Harmonic Maps, 238 — The Global Existence Result, 239 — The Proof of Theorem 6.6, 242 — Finite-Time Blow-Up, 253 — Reverse Bubbling and Nonuniqueness, 257	
Appendix A	263
Sobolev Spaces, 263 — Hölder Spaces, 264 — Imbedding Theorems, 264 — Density Theorem, 265 — Trace and Extension Theorems, 265 — Poincaré Inequality, 266	
Appendix B	268
Schauder Estimates, 268 — L^p -Theory, 268 — Weak Solutions, 269 — A Regularity Result, 269 — Maximum Principle, 271 — Weak Maximum Principle, 272 — Application, 273	
Appendix C	274
Fréchet Differentiability, 274 — Natural Growth Conditions, 276	
References	277
Index	301