

CLASSICAL FIELDS

General Relativity and Gauge Theory

This invaluable book presents gravitation and gauge fields as interrelated topics with a common physical and mathematical foundation, such as gauge theory of gravitation and other fields, giving emphasis to the physicist's point of view.

About half of the material is devoted to Einstein's general relativity theory, and the rest to gauge fields that naturally blend well with gravitation, including spinor formulation, classification of $SU(2)$ gauge fields and null-tetrad formulation of the Yang-Mills field in the presence of gravitation.

The text includes a useful introduction to the physical foundation of the theory of gravitation. It also provides the mathematical theory of the geometry of curved space-times needed to describe Einstein's general relativity theory.

About the Author

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