

Quantum field theory, which started with Paul Dirac's work shortly after the discovery of quantum mechanics, has produced an impressive and important array of results. Quantum electrodynamics, with its extremely accurate and well-tested predictions, and the standard model of electroweak and chromodynamic (nuclear) forces are examples of successful theories. Field theory has also been applied to a variety of phenomena in condensed matter physics, including superconductivity, superfluidity and the quantum Hall effect. The concept of the renormalization group has given us a new perspective on field theory in general and on critical phenomena in particular. At this stage, a strong case can be made that quantum field theory is the mathematical and intellectual framework for describing and understanding all physical phenomena, except possibly for a quantum theory of gravity.

*Quantum Field Theory: A Modern Perspective* presents Professor Nair's view of certain topics in field theory loosely knit together as it grew out of courses on field theory and particle physics taught at Columbia University and the City College of CUNY. The first few chapters contain material that generally goes into any course on quantum field theory, although there are a few nuances of presentation which readers may find to be different from other books. The remaining chapters cover some of the more modern developments over the last three decades, involving topological and geometrical features. These include discussions of anomalies, elements of differential geometry, fields at finite temperature and density, nonperturbative aspects of gauge theories and elements of geometric quantization. Books on relevant mathematical topics which are needed to supplement these chapters are indicated in the bibliography. Professor Nair focuses primarily on developments pertinent to a better understanding of the standard model. *Quantum Field Theory: A Modern Perspective* is designed to serve as a portal to so many more topics of detailed and ongoing research, referring readers to more detailed treatments for many specific topics. The book also contains extensive references, providing readers a more comprehensive perspective on the literature and the historical development of the subject.

V. Parameswaran Nair is Professor of Physics at City College of The City University of New York (CUNY). Professor Nair has held Visiting Professorships at The Abdus Salam International Center for Theoretical Physics, Rockefeller University, Institute for Advanced Study at Princeton, and Massachusetts Institute of Technology.

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