

Contents

Preface	v
Chapter 1. Normed Linear Spaces	1
1.1 Definitions and Examples	1
1.2 Convexity, Convergence, Compactness, Completeness	6
1.3 Continuity, Open Sets, Closed Sets	15
1.4 More About Compactness	19
1.5 Linear Transformations	24
1.6 Zorn's Lemma, Hamel Bases, and the Hahn-Banach Theorem	30
1.7 The Baire Theorem and Uniform Boundedness	40
1.8 The Interior Mapping and Closed Mapping Theorems	47
1.9 Weak Convergence	53
1.10 Reflexive Spaces	58
Chapter 2. Hilbert Spaces	61
2.1 Geometry	61
2.2 Orthogonality and Bases	70
2.3 Linear Functionals and Operators	81
2.4 Spectral Theory	91
2.5 Sturm-Liouville Theory	105
Chapter 3. Calculus in Banach Spaces	115
3.1 The Fréchet Derivative	115
3.2 The Chain Rule and Mean Value Theorems	121
3.3 Newton's Method	125
3.4 Implicit Function Theorems	135
3.5 Extremum Problems and Lagrange Multipliers	145
3.6 The Calculus of Variations	152
Chapter 4. Basic Approximate Methods of Analysis	170
4.1 Discretization	170
4.2 The Method of Iteration	176
4.3 Methods Based on the Neumann Series	186
4.4 Projections and Projection Methods	191
4.5 The Galerkin Method	198
4.6 The Rayleigh-Ritz Method	205
4.7 Collocation Methods	213
4.8 Descent Methods	226
4.9 Conjugate Direction Methods	232
4.10 Methods Based on Homotopy and Continuation	237

Chapter 5. Distributions	246
5.1 Definitions and Examples	246
5.2 Derivatives of Distributions	253
5.3 Convergence of Distributions	257
5.4 Multiplication of Distributions by Functions	260
5.5 Convolutions	268
5.6 Differential Operators	273
5.7 Distributions with Compact Support	280
Chapter 6. The Fourier Transform	287
6.1 Definitions and Basic Properties	287
6.2 The Schwartz Space	294
6.3 The Inversion Theorems	301
6.4 The Plancherel Theorem	305
6.5 Applications of the Fourier Transform	310
6.6 Applications to Partial Differential Equations	318
6.7 Tempered Distributions	321
6.8 Sobolev Spaces	325
Chapter 7. Additional Topics	333
7.1 Fixed-Point Theorems	333
7.2 Selection Theorems	339
7.3 Separation Theorems	342
7.4 The Arzelà-Ascoli Theorems	347
7.5 Compact Operators and the Fredholm Theory	351
7.6 Topological Spaces	361
7.7 Linear Topological Spaces	367
7.8 Analytic Pitfalls	373
Chapter 8. Measure and Integration	381
8.1 Extended Reals, Outer Measures, Measurable Spaces	381
8.2 Measures and Measure Spaces	386
8.3 Lebesgue Measure	391
8.4 Measurable Functions	394
8.5 The Integral for Nonnegative Functions	399
8.6 The Integral, Continued	404
8.7 The L^p -Spaces	409
8.8 The Radon-Nikodym Theorem	413
8.9 Signed Measures	417
8.10 Product Measures and Fubini's Theorem	420
References	429
Index	437
Symbols	443