Linear algebra and matrix theory have long been fundamental tools in mathematical disciplines as well as fertile fields for research. In this book the authors present classical and recent results of matrix analysis that have proved to be important to applied mathematics. Facts about matrices, beyond those found in an elementary linear algebra course, are needed to understand virtually any area of mathematical science, but the necessary material has appeared only sporadically in the literature and in university curricula. As interest in applied mathematics has grown, the need for a text and reference offering a broad selection of topics in matrix theory has become apparent, and this book meets that need.

This volume reflects two concurrent views of matrix analysis. First, it encompasses topics in linear algebra that have arisen out of the needs of mathematical analysis. Second, it is an approach to real and complex linear algebraic problems that does not hesitate to use notions from analysis. Both views are reflected in its choice and treatment of topics.

Matrix Analysis will be welcomed as either an undergraduate or graduate textbook. The authors assume a background in elementary linear algebra and knowledge of rudimentary analytical concepts, beginning with a review of results from elementary linear algebra. Eigenvalues, eigenvectors, and similarity are discussed in the first chapter; the following chapters each treat a major topic in depth. This book will also be useful as a self-contained reference work to a variety of audiences in scientific fields including engineering, statistics, economics, and related disciplines.

"There seems little doubt that the book will become a standard reference for research workers in numerical mathematics" *Computing Reviews*

"The reviewer strongly recommends that those working in either pure or applied linear algebra have this book on their desks" *SIAM Review*

"This will doubtless be the standard text for years to come" *American Scientist*

"On the whole the authors have done an excellent job of supplying linear algebraists and applied mathematicians with a well-organized comprehensive survey, which can serve both as a text and as a reference. The reviewer recommends that everyone working in these fields have this book on his/her desk." *Linear Algebra and its Applications*

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