

Building on the foundations of its predecessor volume, *Matrix Analysis*, this book treats in detail several topics with important applications and of special mathematical interest in matrix theory not included in the previous text. These topics include the field of values, stable matrices and inertia, singular values, matrix equations and Kronecker products, Hadamard products, and matrices and functions. The authors assume a background in elementary linear algebra and knowledge of rudimentary analytical concepts. The book should be welcomed by graduate students and researchers in a variety of mathematical fields both as an advanced text and as a modern reference work.

Also available

Matrix Analysis

In this book the authors present classical and recent results of matrix analysis that have proved to be important to applied mathematics. Facts about matrices, beyond those found in an elementary linear algebra course, are needed to understand virtually any area of mathematical science, but the necessary material has appeared only sporadically in the literature and in university curricula. As interest in applied mathematics has grown, the need for a text and reference offering a broad selection of topics in matrix theory has become apparent, and this book meets that need.

"This will doubtless be the standard text for years to come."

American Scientist

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Contents

Preface		page vii
Chapter 1	The field of values	1
1.0	Introduction	1
1.1	Definitions	5
1.2	Basic properties of the field of values	8
1.3	Convexity	17
1.4	Axiomatization	28
1.5	Location of the field of values	30
1.6	Geometry	48
1.7	Products of matrices	65
1.8	Generalizations of the field of values	77
Chapter 2	Stable matrices and inertia	89
2.0	Motivation	89
2.1	Definitions and elementary observations	91
2.2	Lyapunov's theorem	95
2.3	The Routh-Hurwitz conditions	101
2.4	Generalizations of Lyapunov's theorem	102
2.5	M -matrices, P -matrices, and related topics	112
Chapter 3	Singular value inequalities	134
3.0	Introduction and historical remarks	134
3.1	The singular value decomposition	144
3.2	Weak majorization and doubly substochastic matrices	163
3.3	Basic inequalities for singular values and eigenvalues	170
3.4	Sums of singular values: the Ky Fan k -norms	195
3.5	Singular values and unitarily invariant norms	203

3.6	Sufficiency of Weyl's product inequalities	217
3.7	Inclusion intervals for singular values	223
3.8	Singular value weak majorization for bilinear products	231
Chapter 4 Matrix equations and the Kronecker product		239
4.0	Motivation	239
4.1	Matrix equations	241
4.2	The Kronecker product	242
4.3	Linear matrix equations and Kronecker products	254
4.4	Kronecker sums and the equation $AX + XB = C$	268
4.5	Additive and multiplicative commutators and linear preservers	288
Chapter 5 The Hadamard product		298
5.0	Introduction	298
5.1	Some basic observations	304
5.2	The Schur product theorem	308
5.3	Generalizations of the Schur product theorem	312
5.4	The matrices $A \circ (A^{-1})^T$ and $A \circ A^{-1}$	322
5.5	Inequalities for Hadamard products of general matrices: an overview	332
5.6	Singular values of a Hadamard product: a fundamental inequality	349
5.7	Hadamard products involving nonnegative matrices and M -matrices	356
Chapter 6 Matrices and functions		382
6.0	Introduction	382
6.1	Polynomial matrix functions and interpolation	383
6.2	Nonpolynomial matrix functions	407
6.3	Hadamard matrix functions	449
6.4	Square roots, logarithms, nonlinear matrix equations	459
6.5	Matrices of functions	490
6.6	A chain rule for functions of a matrix	520
Hints for problems		561
References		584
Notation		590
Index		595