

This textbook offers a unified and self-contained introduction to the field of term rewriting. It covers all the basic material (abstract reduction systems, termination, confluence, completion, and combination problems), but also some important and closely connected subjects: universal algebra, unification theory, Gröbner bases and Buchberger's algorithm. The main algorithms are presented both informally and as programs in the functional language Standard ML (an appendix contains a quick and easy introduction to ML). Certain crucial algorithms like unification and congruence closure are covered in more depth and efficient Pascal programs are developed. The book contains many examples and over 170 exercises.

This text is also an ideal reference book for professional researchers: results that have been spread over many conference and journal articles are collected together in a unified notation, detailed proofs of almost all theorems are provided, and each chapter closes with a guide to the literature.

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