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This computational fluid dynamics (CFD) book is *truly for beginners*. If you have never studied CFD before, if you have never worked in the area, and if you have no real idea as to what the discipline is all about, then this book is for you. Absolutely no prior knowledge of CFD is assumed on your part—only your desire to learn something about the subject is taken for granted.

The author's single-minded purpose in writing this book is to provide a simple, engaging, and motivational approach toward presenting the subject to the reader who is learning about CFD for the first time. In the workplace, CFD is today a technologically sophisticated discipline. In turn, in the universities it is generally considered to be a graduate-level subject; the existing textbooks and most of the professional development short courses are pitched at the graduate level. The present book is a *precursor* to these activities. It is intended to "break the ice" for the reader. This book is *unique* in that it is intended to be read and mastered *before* you do any of the other existing textbooks in the field, *before* you take any of the short courses in the discipline, and *before* you endeavor to read the existing graduate texts. The hallmarks of the present book are *simplicity* and *motivation*. It is intended to prepare you for the more sophisticated presentations elsewhere—to give you a good appreciation for the basic philosophy and ideas which will then make the more sophisticated presentations more meaningful to you later on. The educational level and the prior background in fluid dynamics assumed in this book are equivalent to those of a college senior in engineering or physical science. However, the book is targeted primarily for use as a one-semester, senior-level course which may also be useful in a preliminary, first-level graduate course.

There are no role models for a book on CFD at the undergraduate level; when you ask ten different people about what form such a book should take, you get ten different answers. This book is the author's answer, as imperfect as it may be, developed after many years of thought and teaching experience. Of course, to accomplish the goals stated above, the author has made some hard choices in picking up and dropping the material in this book. It is *not* a state-of-the-art treatment of the highly sophisticated CFD of today. Such a treatment would blow the uninitiated reader completely out of the water. This author knows; he has seen it happen over