

This text is a clearly written and complete introductory study of differential equations and their applications. Suitable for a one- or two-semester course on the subject, the material is fully comprehensible to anyone who has studied college-level calculus. This book distinguishes itself from other differential equation texts through its application of the subject matter to fascinating events and its incorporation of relatively recent developments in the field. Some applications include

- a proof that the painting, "Disciples at Emmaus," bought by the Rembrandt Society of Belgium was a modern forgery;
- a mathematical explanation of the Tacoma Bridge disaster;
- a model of the blood glucose regulatory system which leads to a criterion for the diagnosis of diabetes;
- an explanation of why the predator portion (sharks, skates, rays, etc.) of all fish caught in the port of Fiume, Italy, rose dramatically during the years of World War I;
- a mathematical verification of Darwin's law that "the more similar two species are, the fiercer is the struggle for existence between them."

An introduction to bifurcation theory, computer programs in C, Pascal, and Fortran, and many original and challenging exercises contribute to the quality of this text.



## Chapter 1

<b>First-order differential equations</b>	<b>1</b>
1.1 Introduction	1
1.2 First-order linear differential equations	2
1.3 The Van Meegeren art forgeries	11
1.4 Separable equations	20
1.5 Population models	26
1.6 The spread of technological innovations	39
1.7 An atomic waste disposal problem	46
1.8 The dynamics of tumor growth, mixing problems, and orthogonal trajectories	52
1.9 Exact equations, and why we cannot solve very many differential equations	58
1.10 The existence–uniqueness theorem; Picard iteration	67
1.11 Finding roots of equations by iteration	81
1.11.1 Newton’s method	87
1.12 Difference equations, and how to compute the interest due on your student loans	91
1.13 Numerical approximations; Euler’s method	96
1.13.1 Error analysis for Euler’s method	100
1.14 The three term Taylor series method	107
1.15 An improved Euler method	109
1.16 The Runge–Kutta method	112
1.17 What to do in practice	116

## Chapter 2

<b>Second-order linear differential equations</b>	<b>127</b>
2.1 Algebraic properties of solutions	127
2.2 Linear equations with constant coefficients	138
2.2.1 Complex roots	141
2.2.2 Equal roots; reduction of order	145
2.3 The nonhomogeneous equation	151
2.4 The method of variation of parameters	153
2.5 The method of judicious guessing	157
2.6 Mechanical vibrations	165
2.6.1 The Tacoma Bridge disaster	173
2.6.2 Electrical networks	175
2.7 A model for the detection of diabetes	178
2.8 Series solutions	185
2.8.1 Singular points, Euler equations	198
2.8.2 Regular singular points, the method of Frobenius	203
2.8.3 Equal roots, and roots differing by an integer	219
2.9 The method of Laplace transforms	225
2.10 Some useful properties of Laplace transforms	233
2.11 Differential equations with discontinuous right-hand sides	238
2.12 The Dirac delta function	243
2.13 The convolution integral	251
2.14 The method of elimination for systems	257
2.15 Higher-order equations	259

## Chapter 3

<b>Systems of differential equations</b>	<b>264</b>
3.1 Algebraic properties of solutions of linear systems	264
3.2 Vector spaces	273
3.3 Dimension of a vector space	279
3.4 Applications of linear algebra to differential equations	291
3.5 The theory of determinants	297
3.6 Solutions of simultaneous linear equations	310
3.7 Linear transformations	320
3.8 The eigenvalue–eigenvector method of finding solutions	333
3.9 Complex roots	341
3.10 Equal roots	345
3.11 Fundamental matrix solutions; $e^{At}$	355
3.12 The nonhomogeneous equation; variation of parameters	360
3.13 Solving systems by Laplace transforms	368

## Chapter 4

<b>Qualitative theory of differential equations</b>	<b>372</b>
4.1 Introduction	372
4.2 Stability of linear systems	378

4.3	Stability of equilibrium solutions	385
4.4	The phase-plane	394
4.5	Mathematical theories of war	398
4.5.1	L. F. Richardson's theory of conflict	398
4.5.2	Lanchester's combat models and the battle of Iwo Jima	405
4.6	Qualitative properties of orbits	414
4.7	Phase portraits of linear systems	418
4.8	Long time behavior of solutions; the Poincaré–Bendixson Theorem	428
4.9	Introduction to bifurcation theory	437
4.10	Predator-prey problems; or why the percentage of sharks caught in the Mediterranean Sea rose dramatically during World War I	443
4.11	The principle of competitive exclusion in population biology	451
4.12	The Threshold Theorem of epidemiology	458
4.13	A model for the spread of gonorrhea	465

## Chapter 5

## Separation of variables and Fourier series 476

5.1	Two point boundary-value problems	476
5.2	Introduction to partial differential equations	481
5.3	The heat equation; separation of variables	483
5.4	Fourier series	487
5.5	Even and odd functions	493
5.6	Return to the heat equation	498
5.7	The wave equation	503
5.8	Laplace's equation	508

## Chapter 6

## Sturm–Liouville boundary value problems 514

6.1	Introduction	514
6.2	Inner product spaces	515
6.3	Orthogonal bases, Hermitian operators	526
6.4	Sturm–Liouville theory	533

## Appendix A

Some simple facts concerning functions of several variables	545
--	-----

## Appendix B

Sequences and series	547
----------------------	-----

## Appendix C

C Programs	549
------------	-----

## Contents

<b>Answers to odd-numbered exercises</b>	<b>557</b>
<b>Index</b>	<b>575</b>