Fractal Concepts in Surface Growth

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The use of fractal concepts in understanding various growth phenomena, such as molecular beam epitaxy (MBE) or fluid flow in porous media, is increasingly important these days. This book introduces the basic models and concepts that are necessary to understand in a pedagogical way the various growth processes leading to rough interfaces. The text will be accessible to readers not familiar with the field.

Nature provides a large number of rough surfaces and interfaces. Similarly, rough surfaces are regularly observed in the laboratory during various technologically important growth technologies, such as MBE. In an attempt to understand the origin of the roughening phenomena, several computer models and theoretical approaches have recently been developed. The principal goal of this book is to describe the basic models and theories as well as the principles one uses to develop a model for a particular growth process. Furthermore, having described a particular growth model, the authors show how one can address and answer questions such as whether the surface will be rough, how rough it will be, and how to characterize this roughness. Having introduced the basic methods and tools needed to study a growth model, the authors discuss in detail two classes of phenomena: fluid flow in a porous medium and molecular beam epitaxy. In both cases, in addition to the models and analytical approaches, the authors describe the relevant experimental results as well.

This text contains homework problems at the ends of chapters, and will be invaluable for advanced undergraduates, graduate students and researchers in physics, materials science, chemistry and engineering, and especially those interested in condensed matter physics and surface growth.

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