

This book provides an overview of the theory and application of linear and nonlinear mixed-effects models in the analysis of grouped data, such as longitudinal data, repeated measures, and multilevel data. A unified model-building strategy for both linear and nonlinear models is presented and applied to the analysis of over 20 real datasets from a wide variety of areas, including pharmacokinetics, agriculture, and manufacturing. A strong emphasis is placed on the use of graphical displays at the various phases of the model-building process, starting with exploratory plots of the data and concluding with diagnostic plots to assess the adequacy of a fitted model. Over 170 figures are included in the book.

The NLME library for analyzing mixed-effects models in S and S-PLUS, developed by the authors, provides the underlying software for implementing the methods presented in the text, and is described and illustrated in detail throughout the book.

The balanced mix of real data examples, modeling software, and theory makes this book a useful reference for practitioners using mixed-effects models in their data analyses. It can also be used as a text for a one-semester graduate-level applied course in mixed-effects models. Researchers in statistical computing will also find this book appealing for its presentation of novel and efficient computational methods for fitting linear and nonlinear mixed-effects models.

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