Contents

List of figures *page* x
List of tables xii
Preface xiii

1 Introduction 1
1.1 Main motivations and scope 1
1.2 Basic definitions and examples 2
1.3 Conventional notation 9

2 Properties for aggregation 11
2.1 Introduction 11
2.2 Elementary mathematical properties 12
2.3 Grouping-based properties 31
2.4 Invariance properties 41
2.5 Further properties 49

3 Conjunctive and disjunctive aggregation functions 56
3.1 Preliminaries and general notes 56
3.2 Generated conjunctive aggregation functions 59
3.3 Triangular norms and related conjunctive aggregation functions 64
3.4 Copulas and quasi-copulas 88
3.5 Disjunctive aggregation functions 100
3.6 Uninorms 106
3.7 Nullnorms 115
3.8 More aggregation functions related to t-norms 119
3.9 Restricted distributivity 123

4 Means and averages 130
4.1 Introduction and definitions 130
4.2 Quasi-arithmetic means 132

4.3 Generalizations of quasi-arithmetic means 139
4.4 Associative means 161
4.5 Means constructed from a mean value property 163
4.6 Constructing means 166
4.7 Further extended means 168

5 Aggregation functions based on nonadditive integrals 171
5.1 Introduction 171
5.2 Set functions, capacities, and games 172
5.3 Some linear transformations of set functions 177
5.4 The Choquet integral 181
5.5 The Sugeno integral 207
5.6 Other integrals 227

6 Construction methods 234
6.1 Introduction 234
6.2 Transformed aggregation functions 234
6.3 Composed aggregation 242
6.4 Weighted aggregation functions 247
6.5 Some other aggregation-based construction methods 252
6.6 Aggregation functions based on minimal dissimilarity 257
6.7 Ordinal sums of aggregation functions 261
6.8 Extensions to aggregation functions 266

7 Aggregation on specific scale types 272
7.1 Introduction 272
7.2 Ratio scales 273
7.3 Difference scales 280
7.4 Interval scales 284
7.5 Log-ratio scales 289

8 Aggregation on ordinal scales 292
8.1 Introduction 292
8.2 Order invariant subsets 293
8.3 Lattice polynomial functions and some of their properties 296
8.4 Ordinal scale invariant functions 300
8.5 Comparison meaningful functions on a single ordinal scale 304
8.6 Comparison meaningful functions on independent ordinal scales 308
8.7 Aggregation on finite chains by chain independent functions 310

9 Aggregation on bipolar scales 317
9.1 Introduction 317
9.2 Associative bipolar operators 319

9.3 Minimum and maximum on symmetrized linearly ordered sets 325
9.4 Separable aggregation functions 332
9.5 Integral-based aggregation functions 334

10 Behavioral analysis of aggregation functions 348
10.1 Introduction 348
10.2 Expected values and distribution functions 348
10.3 Importance indices 361
10.4 Interaction indices 367
10.5 Maximum improving index 370
10.6 Tolerance indices 372
10.7 Measures of arguments contribution and involvement 378

11 Identification of aggregation functions 382
11.1 Introduction 382
11.2 General formulation 383
11.3 The case of parametrized families of aggregation functions 386
11.4 The case of generated aggregation functions 388
11.5 The case of integral-based aggregation functions 391
11.6 Available software 396

Appendix A: Aggregation of infinitely many arguments 397
A.1 Introduction 397
A.2 Infinitary aggregation functions on sequences 397
A.3 General aggregation of infinite number of inputs 405

Appendix B: Examples and applications 410
B.1 Main domains of applications 410
B.2 A specific application: mixture of uncertainty measures 414

List of symbols 420
References 428
Index 454