

LONDON MATHEMATICAL SOCIETY LECTURE NOTE SERIES

Edited by PROFESSOR N. J. HITCHIN

*Mathematical Institute, University of Oxford, 24–29 St Giles, Oxford OX1 3LB,
United Kingdom*

with the assistance of

S. Donkin (*London*)

I. Fesenko (*Nottingham*)

J. Roe (*Pennsylvania*)

E. Süli (*Oxford*)

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A Quantum Groups Primer

Shahn Majid

This book provides a self-contained introduction to quantum groups as algebraic objects. Based on the author's lecture notes for a Part III pure mathematics course at Cambridge University, it is suitable for use as a textbook for graduate courses in quantum groups or as a supplement to modern courses in advanced algebra. The book assumes a background knowledge of basic algebra and linear algebra. Some familiarity with semisimple Lie algebras would also be helpful. The book is aimed as a primer for mathematicians interested in quantum groups, algebraic groups, knot theory and noncommutative geometry, but will also be useful for mathematical physicists.

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