# CONTENTS


**Preface** x

**1 Introduction** 1

**I The empirical study of networks** 15

**2 Technological networks** 17
- 2.1 The Internet . . . 18
- 2.2 The telephone network . . . 28
- 2.3 Power grids . . . 31
- 2.4 Transportation networks . . . 32
- 2.5 Delivery and distribution networks . . . 33

**3 Social networks** 36
- 3.1 The empirical study of social networks . . . 36
- 3.2 Interviews and questionnaires . . . 39
- 3.3 Direct observation . . . 46
- 3.4 Data from archival or third-party records . . . 47
- 3.5 Affiliation networks . . . 53
- 3.6 The small-world experiment . . . 54
- 3.7 Snowball sampling, contact tracing, and random walks . . . 58

**4 Networks of information** 63
- 4.1 The World Wide Web . . . 63
- 4.2 Citation networks . . . 67
- 4.3 Other information networks . . . 72

**5 Biological networks** 78
- 5.1 Biochemical networks . . . 78
- 5.2 Neural networks . . . 94
- 5.3 Ecological networks . . . 99

**II Fundamentals of network theory** **107**

**6 Mathematics of networks** **109**
6.1 Networks and their representation . . . . . 109
6.2 The adjacency matrix . . . . . 110
6.3 Weighted networks . . . . . 112
6.4 Directed networks . . . . . 114
6.5 Hypergraphs . . . . . 122
6.6 Bipartite networks . . . . . 123
6.7 Trees . . . . . 127
6.8 Planar networks . . . . . 129
6.9 Degree . . . . . 133
6.10 Paths . . . . . 136
6.11 Components . . . . . 142
6.12 Independent paths, connectivity, and cut sets . . . . . 145
6.13 The graph Laplacian . . . . . 152
6.14 Random walks . . . . . 157

**7 Measures and metrics** **168**
7.1 Degree centrality . . . . . 168
7.2 Eigenvector centrality . . . . . 169
7.3 Katz centrality . . . . . 172
7.4 PageRank . . . . . 175
7.5 Hubs and authorities . . . . . 178
7.6 Closeness centrality . . . . . 181
7.7 Betweenness centrality . . . . . 185
7.8 Groups of vertices . . . . . 193
7.9 Transitivity . . . . . 198
7.10 Reciprocity . . . . . 204
7.11 Signed edges and structural balance . . . . . 206
7.12 Similarity . . . . . 211
7.13 Homophily and assortative mixing . . . . . 220

**8 The large-scale structure of networks** **235**
8.1 Components . . . . . 235
8.2 Shortest paths and the small-world effect . . . . . 241
8.3 Degree distributions . . . . . 243
8.4 Power laws and scale-free networks . . . . . 247
8.5 Distributions of other centrality measures . . . . . 261
8.6 Clustering coefficients . . . . . 262

8.7 Assortative mixing . . . . . 266

**III Computer algorithms** **273**

**9 Basic concepts of algorithms** **275**
9.1 Running time and computational complexity . . . . . 278
9.2 Storing network data . . . . . 282
9.3 The adjacency matrix . . . . . 283
9.4 The adjacency list . . . . . 286
9.5 Trees . . . . . 290
9.6 Other network representations . . . . . 298
9.7 Heaps . . . . . 301

**10 Fundamental network algorithms** **308**
10.1 Algorithms for degrees and degree distributions . . . . . 308
10.2 Clustering coefficients . . . . . 310
10.3 Shortest paths and breadth-first search . . . . . 315
10.4 Shortest paths in networks with varying edge lengths . . . . . 329
10.5 Maximum flows and minimum cuts . . . . . 333

**11 Matrix algorithms and graph partitioning** **345**
11.1 Leading eigenvectors and eigenvector centrality . . . . . 345
11.2 Dividing networks into clusters . . . . . 354
11.3 Graph partitioning . . . . . 358
11.4 The Kernighan–Lin algorithm . . . . . 360
11.5 Spectral partitioning . . . . . 364
11.6 Community detection . . . . . 371
11.7 Simple modularity maximization . . . . . 373
11.8 Spectral modularity maximization . . . . . 375
11.9 Division into more than two groups . . . . . 378
11.10 Other modularity maximization methods . . . . . 380
11.11 Other algorithms for community detection . . . . . 382

**IV Network models** **395**

**12 Random graphs** **397**
12.1 Random graphs . . . . . 398
12.2 Mean number of edges and mean degree . . . . . 400
12.3 Degree distribution . . . . . 401

12.4 Clustering coefficient . . . . . 402
12.5 Giant component . . . . . 403
12.6 Small components . . . . . 408
12.7 Path lengths . . . . . 419
12.8 Problems with the random graph . . . . . 423

**13 Random graphs with general degree distributions** **428**
13.1 Generating functions . . . . . 429
13.2 The configuration model . . . . . 434
13.3 Excess degree distribution . . . . . 445
13.4 Clustering coefficient . . . . . 449
13.5 Generating functions for degree distributions . . . . . 450
13.6 Number of second neighbors of a vertex . . . . . 451
13.7 Generating functions for the small components . . . . . 456
13.8 Giant component . . . . . 460
13.9 Size distribution for small components . . . . . 465
13.10 Power-law degree distributions . . . . . 470
13.11 Directed random graphs . . . . . 473

**14 Models of network formation** **486**
14.1 Preferential attachment . . . . . 487
14.2 The model of Barabási and Albert . . . . . 500
14.3 Further properties of preferential attachment models . . . . . 503
14.4 Extensions of preferential attachment models . . . . . 514
14.5 Vertex copying models . . . . . 534
14.6 Network optimization models . . . . . 541

**15 Other network models** **552**
15.1 The small-world model . . . . . 552
15.2 Exponential random graphs . . . . . 565

**V Processes on networks** **589**

**16 Percolation and network resilience** **591**
16.1 Percolation . . . . . 592
16.2 Uniform random removal of vertices . . . . . 594
16.3 Non-uniform removal of vertices . . . . . 609
16.4 Percolation in real-world networks . . . . . 615
16.5 Computer algorithms for percolation . . . . . 616

**17 Epidemics on networks** **627**
17.1 Models of the spread of disease . . . . . 627
17.2 The SI model . . . . . 628
17.3 The SIR model . . . . . 631
17.4 The SIS model . . . . . 636
17.5 The SIRS model . . . . . 637
17.6 Epidemic models on networks . . . . . 639
17.7 Late-time properties of epidemics on networks . . . . . 640
17.8 Late-time properties of the SIR model . . . . . 642
17.9 Time-dependent properties of epidemics on networks . . . . . 648
17.10 Time-dependent properties of the SI model . . . . . 648
17.11 Time-dependent properties of the SIR model . . . . . 661
17.12 Time-dependent properties of the SIS model . . . . . 669

**18 Dynamical systems on networks** **676**
18.1 Dynamical systems . . . . . 677
18.2 Dynamics on networks . . . . . 686
18.3 Dynamics with more than one variable per vertex . . . . . 695
18.4 Synchronization . . . . . 701

**19 Network search** **705**
19.1 Web search . . . . . 705
19.2 Searching distributed databases . . . . . 709
19.3 Message passing . . . . . 713

**References** **727**

**Index** **740**