

Contents

Preface to the Second Edition.....	xiii
Preface to the First Edition.....	xix
1 Introduction	1
1.1 Factor Analysis and Structural Theories	1
1.2 Brief History of Factor Analysis as a Linear Model.....	3
1.3 Example of Factor Analysis	12
2 Mathematical Foundations for Factor Analysis	17
2.1 Introduction	17
2.2 Scalar Algebra.....	17
2.2.1 Fundamental Laws of Scalar Algebra	18
2.2.1.1 Rules of Signs.....	18
2.2.1.2 Rules for Exponents	19
2.2.1.3 Solving Simple Equations	19
2.3 Vectors	20
2.3.1 n -Tuples as Vectors.....	22
2.3.1.1 Equality of Vectors.....	22
2.3.2 Scalars and Vectors.....	23
2.3.3 Multiplying a Vector by a Scalar	23
2.3.4 Addition of Vectors	24
2.3.5 Scalar Product of Vectors	24
2.3.6 Distance between Vectors	25
2.3.7 Length of a Vector	26
2.3.8 Another Definition for Scalar Multiplication	27
2.3.9 Cosine of the Angle between Vectors	27
2.3.10 Projection of a Vector onto Another Vector	29
2.3.11 Types of Special Vectors	30
2.3.12 Linear Combinations	31
2.3.13 Linear Independence	32
2.3.14 Basis Vectors.....	32
2.4 Matrix Algebra	32
2.4.1 Definition of a Matrix	32
2.4.2 Matrix Operations	33
2.4.2.1 Equality.....	34
2.4.2.2 Multiplication by a Scalar	34
2.4.2.3 Addition.....	34
2.4.2.4 Subtraction	35
2.4.2.5 Matrix Multiplication	35

2.4.3	Identity Matrix.....	37
2.4.4	Scalar Matrix	38
2.4.5	Diagonal Matrix	39
2.4.6	Upper and Lower Triangular Matrices	39
2.4.7	Null Matrix.....	40
2.4.8	Transpose Matrix.....	40
2.4.9	Symmetric Matrices	41
2.4.10	Matrix Inverse.....	41
2.4.11	Orthogonal Matrices	42
2.4.12	Trace of a Matrix.....	43
2.4.13	Invariance of Traces under Cyclic Permutations	43
2.5	Determinants	44
2.5.1	Minors of a Matrix	46
2.5.2	Rank of a Matrix	47
2.5.3	Cofactors of a Matrix	47
2.5.4	Expanding a Determinant by Cofactors	48
2.5.5	Adjoint Matrix	48
2.5.6	Important Properties of Determinants.....	49
2.5.7	Simultaneous Linear Equations	50
2.6	Treatment of Variables as Vectors	51
2.6.1	Variables in Finite Populations	51
2.6.2	Variables in Infinite Populations.....	53
2.6.3	Random Vectors of Random Variables.....	56
2.7	Maxima and Minima of Functions	58
2.7.1	Slope as the Indicator of a Maximum or Minimum.....	59
2.7.2	Index for Slope.....	59
2.7.3	Derivative of a Function.....	60
2.7.4	Derivative of a Constant	62
2.7.5	Derivative of Other Functions.....	62
2.7.6	Partial Differentiation	64
2.7.7	Maxima and Minima of Functions of Several Variables....	65
2.7.8	Constrained Maxima and Minima	67
3	Composite Variables and Linear Transformations.....	69
3.1	Introduction	69
3.1.1	Means and Variances of Variables	69
3.1.1.1	Correlation and Causation.....	71
3.2	Composite Variables	72
3.3	Unweighted Composite Variables.....	73
3.3.1	Mean of an Unweighted Composite	73
3.3.2	Variance of an Unweighted Composite	73
3.3.3	Covariance and Correlation between Two Composites.....	77
3.3.4	Correlation of an Unweighted Composite with a Single Variable.....	78

3.3.5	Correlation between Two Unweighted Composites.....	80
3.3.6	Summary Concerning Unweighted Composites.....	83
3.4	Differentially Weighted Composites.....	83
3.4.1	Correlation between a Differentially Weighted Composite and Another Variable	83
3.4.2	Correlation between Two Differentially Weighted Composites.....	84
3.5	Matrix Equations.....	84
3.5.1	Random Vectors, Mean Vectors, Variance–Covariance Matrices, and Correlation Matrices	84
3.5.2	Sample Equations.....	86
3.5.3	Composite Variables in Matrix Equations	88
3.5.4	Linear Transformations	89
3.5.5	Some Special, Useful Linear Transformations	91
4	Multiple and Partial Correlations	93
4.1	Multiple Regression and Correlation.....	93
4.1.1	Minimizing the Expected Squared Difference between a Composite Variable and an External Variable	93
*4.1.2	Deriving the Regression Weight Matrix for Multivariate Multiple Regression.....	95
4.1.3	Matrix Equations for Multivariate Multiple Regression	97
4.1.4	Squared Multiple Correlations.....	98
4.1.5	Correlations between Actual and Predicted Criteria	99
4.2	Partial Correlations.....	100
4.3	Determinantal Formulas	102
4.3.1	Multiple-Correlation Coefficient.....	103
4.3.2	Formulas for Partial Correlations	104
4.4	Multiple Correlation in Terms of Partial Correlation	104
4.4.1	Matrix of Image Regression Weights.....	105
4.4.2	Meaning of Multiple Correlation.....	107
4.4.3	Yule's Equation for the Error of Estimate	109
4.4.4	Conclusions.....	110
5	Multivariate Normal Distribution.....	113
5.1	Introduction	113
5.2	Univariate Normal Density Function	113
5.3	Multivariate Normal Distribution	114
5.3.1	Bivariate Normal Distribution	115
5.3.2	Properties of the Multivariate Normal Distribution	116
*5.4	Maximum-Likelihood Estimation.....	118
5.4.1	Notion of Likelihood	118
5.4.2	Sample Likelihood	119

5.4.3	Maximum-Likelihood Estimates	119
5.4.4	Multivariate Case	124
5.4.4.1	Distribution of \bar{y} and S	128
6	Fundamental Equations of Factor Analysis	129
6.1	Analysis of a Variable into Components	129
6.1.1	Components of Variance	132
6.1.2	Variance of a Variable in Terms of Its Factors	133
6.1.3	Correlation between Two Variables in Terms of Their Factors.....	134
6.2	Use of Matrix Notation in Factor Analysis.....	135
6.2.1	Fundamental Equation of Factor Analysis	135
6.2.2	Fundamental Theorem of Factor Analysis	136
6.2.3	Factor-Pattern and Factor-Structure Matrices.....	137
7	Methods of Factor Extraction	139
7.1	Rationale for Finding Factors and Factor Loadings.....	139
7.1.1	General Computing Algorithm for Finding Factors.....	140
7.2	Diagonal Method of Factoring	145
7.3	Centroid Method of Factoring	147
7.4	Principal-Axes Methods.....	147
7.4.1	Hotelling's Iterative Method	151
7.4.2	Further Properties of Eigenvectors and Eigenvalues	154
7.4.3	Maximization of Quadratic Forms for Points on the Unit Sphere	156
7.4.4	Diagonalizing the R Matrix into Its Eigenvalues	158
7.4.5	Jacobi Method	159
7.4.6	Powers of Square Symmetric Matrices	164
7.4.7	Factor-Loading Matrix from Eigenvalues and Eigenvectors	165
8	Common-Factor Analysis.....	167
8.1	Preliminary Considerations.....	167
8.1.1	Designing a Factor Analytic Study	168
8.2	First Stages in the Factor Analysis.....	169
8.2.1	Concept of Minimum Rank	170
8.2.2	Systematic Lower-Bound Estimates of Communalities.....	175
8.2.3	Congruence Transformations	176
8.2.4	Sylvester's Law of Inertia	176
8.2.5	Eigenvector Transformations	177
8.2.6	Guttman's Lower Bounds for Minimum Rank.....	177
8.2.7	Preliminary Theorems for Guttman's Bounds.....	178

8.2.8	Proof of the First Lower Bound.....	181
8.2.9	Proof of the Third Lower Bound.....	181
8.2.10	Proof of the Second Lower Bound.....	184
8.2.11	Heuristic Rules of Thumb for the Number of Factors	185
8.2.11.1	Kaiser's Eigenvalues-Greater-Than-One Rule	186
8.2.11.2	Cattell's Scree Criterion	186
8.2.11.3	Parallel Analysis	188
8.3	Fitting the Common-Factor Model to a Correlation Matrix	192
8.3.1	Least-Squares Estimation of the Exploratory Common-Factor Model.....	193
8.3.2	Assessing Fit	197
8.3.3	Example of Least-Squares Common-Factor Analysis.....	197
8.3.4	Maximum-Likelihood Estimation of the Exploratory Common-Factor Model	199
*8.3.4.1	Maximum-Likelihood Estimation Obtained Using Calculus	202
8.3.5	Maximum-Likelihood Estimates	206
*8.3.6	Fletcher-Powell Algorithm.....	207
*8.3.7	Applying the Fletcher-Powell Algorithm to Maximum-Likelihood Exploratory Factor Analysis.....	210
8.3.8	Testing the Goodness of Fit of the Maximum-Likelihood Estimates	212
8.3.9	Optimality of Maximum-Likelihood Estimators.....	214
8.3.10	Example of Maximum-Likelihood Factor Analysis	215
9	Other Models of Factor Analysis.....	217
9.1	Introduction	217
9.2	Component Analysis	217
9.2.1	Principal-Components Analysis	219
9.2.2	Selecting Fewer Components than Variables.....	220
9.2.3	Determining the Reliability of Principal Components ...	222
9.2.4	Principal Components of True Components.....	224
9.2.5	Weighted Principal Components	226
9.3	Image Analysis	230
9.3.1	Partial-Image Analysis	231
9.3.2	Image Analysis and Common-Factor Analysis	237
9.3.3	Partial-Image Analysis as Approximation of Common-Factor Analysis.....	244
9.4	Canonical-Factor Analysis	245
9.4.1	Relation to Image Analysis	249
9.4.2	Kaiser's Rule for the Number of Harris Factors.....	253
9.4.3	Quickie, Single-Pass Approximation for Common-Factor Analysis.....	253

181 9.5 Problem of Doublet Factors.....	253
181 9.5.1 Butler's Descriptive-Factor-Analysis Solution	254
181 9.5.2 Model That Includes Doublets Explicitly	258
281 9.6 Metric Invariance Properties	262
6 9.7 Image-Factor Analysis.....	263
6 9.7.1 Testing Image Factors for Significance.....	264
881 9.8 Psychometric Inference in Factor Analysis	265
881 9.8.1 Alpha Factor Analysis	270
881 9.8.2 Communality in a Universe of Tests	271
881 9.8.3 Consequences for Factor Analysis.....	274
101 10 Factor Rotation	275
101 10.1 Introduction	275
101 10.2 Thurstone's Concept of a Simple Structure.....	276
101 10.2.1 Implementing the Simple-Structure Concept	280
101 10.2.2 Question of Correlated Factors	282
101 10.3 Oblique Graphical Rotation	286
101 11 Orthogonal Analytic Rotation	301
101 11.1 Introduction	301
101 11.2 Quartimax Criterion	302
101 11.3 Varimax Criterion.....	310
101 11.4 Transvarimax Methods.....	312
101 11.4.1 Parsimax	313
101 11.5 Simultaneous Orthogonal Varimax and Parsimax	315
101 11.5.1 Gradient Projection Algorithm.....	323
101 12 Oblique Analytic Rotation	325
101 12.1 General	325
101 12.1.1 Distinctness of the Criteria in Oblique Rotation.....	325
101 12.2 Oblimin Family	326
101 12.2.1 Direct Oblimin by Planar Rotations	328
101 12.3 Harris-Kaiser Oblique Transformations	332
101 12.4 Weighted Oblique Rotation	336
101 12.5 Oblique Procrustean Transformations	341
101 12.5.1 Promax Oblique Rotation	342
101 12.5.2 Rotation to a Factor-Pattern Matrix Approximating a Given Target Matrix.....	343
101 12.5.3 Promaj	343
101 12.5.4 Promin	345
101 12.6 Gradient-Projection-Algorithm Synthesis.....	348
101 12.6.1 Gradient-Projection Algorithm	348
101 12.6.2 Jennrich's Use of the GPA	351
101 12.6.2.1 Gradient-Projection Algorithm	353
101 12.6.2.2 Quartimin.....	353

12.6.2.3	Oblimin Rotation.....	354
12.6.2.4	Least-Squares Rotation to a Target Matrix	357
12.6.2.5	Least-Squares Rotation to a Partially Specified Target Pattern Matrix.....	357
12.6.3	Simplimax	357
12.7	Rotating Using Component Loss Functions	360
12.8	Conclusions.....	366
13	Factor Scores and Factor Indeterminacy	369
13.1	Introduction	369
13.2	Scores on Component Variables	370
13.2.1	Component Scores in Canonical-Component Analysis and Image Analysis	373
13.2.1.1	Canonical-Component Analysis.....	373
13.2.1.2	Image Analysis.....	374
13.3	Indeterminacy of Common-Factor Scores	375
13.3.1	Geometry of Correlational Indeterminacy	377
13.4	Further History of Factor Indeterminacy	380
13.4.1	Factor Indeterminacy from 1970 to 1980.....	384
13.4.1.1	"Infinite Domain" Position	392
13.4.2	Researchers with Well-Defined Concepts of Their Domains	395
13.4.2.1	Factor Indeterminacy from 1980 to 2000	397
13.5	Other Estimators of Common Factors	399
13.5.1	Least Squares	400
13.5.2	Bartlett's Method.....	401
13.5.3	Evaluation of Estimation Methods	403
14	Factorial Invariance.....	405
14.1	Introduction	405
14.2	Invariance under Selection of Variables	405
14.3	Invariance under Selection of Experimental Populations	408
14.3.1	Effect of Univariate Selection	408
14.3.2	Multivariate Case	412
14.3.3	Factorial Invariance in Different Experimental Populations	414
14.3.4	Effects of Selection on Component Analysis.....	418
14.4	Comparing Factors across Populations	419
14.4.1	Preliminary Requirements for Comparing Factor Analyses	420
14.4.2	Inappropriate Comparisons of Factors	421
14.4.3	Comparing Factors from Component Analyses	422
14.4.4	Contrasting Experimental Populations across Factors.....	423
14.4.5	Limitations on Factorial Invariance.....	424

15 Confirmatory Factor Analysis	427
15.1 Introduction	427
15.1.1 Abduction, Deduction, and Induction	428
15.1.2 Science as the Knowledge of Objects	429
15.1.3 Objects as Invariants in the Perceptual Field	431
15.1.4 Implications for Factor Analysis	433
15.2 Example of Confirmatory Factor Analysis.....	434
15.3 Mathematics of Confirmatory Factor Analysis.....	440
15.3.1 Specifying Hypotheses.....	440
15.3.2 Identification.....	441
15.3.3 Determining Whether Parameters and Models Are Identified	444
15.3.4 Identification of Metrics	450
15.3.5 Discrepancy Functions	452
15.3.6 Estimation by Minimizing Discrepancy Functions.....	454
15.3.7 Derivatives of Elements of Matrices.....	454
15.3.8 Maximum-Likelihood Estimation in Confirmatory Factor Analysis	457
15.3.9 Least-Squares Estimation.....	461
15.3.10 Generalized Least-Squares Estimation	463
15.3.11 Implementing the Quasi-Newton Algorithm	463
15.3.12 Avoiding Improper Solutions.....	465
15.3.13 Statistical Tests.....	466
15.3.14 What to Do When Chi-Square Is Significant.....	467
15.3.15 Approximate Fit Indices.....	469
15.4 Designing Confirmatory Factor Analysis Models.....	473
15.4.1 Restricted versus Unrestricted Models	473
15.4.2 Use for Unrestricted Model	475
15.4.3 Measurement Model.....	476
15.4.4 Four-Step Procedure for Evaluating a Model	477
15.5 Some Other Applications	477
15.5.1 Faceted Classification Designs	477
15.5.2 Multirater–Multioccasion Studies	478
15.5.3 Multitrait–Multimethod Covariance Matrices.....	483
15.6 Conclusion	489
References	493
Author Index	505
Subject Index	509