Contents

Preface xii

Advertisement xiv

1 ■ Vector Analysis 1

1.1 Vector Algebra 1
- 1.1.1 Vector Operations 1
- 1.1.2 Vector Algebra: Component Form 4
- 1.1.3 Triple Products 7
- 1.1.4 Position, Displacement, and Separation Vectors 8
- 1.1.5 How Vectors Transform 10

1.2 Differential Calculus 13
- 1.2.1 "Ordinary" Derivatives 13
- 1.2.2 Gradient 13
- 1.2.3 The Del Operator 16
- 1.2.4 The Divergence 17
- 1.2.5 The Curl 18
- 1.2.6 Product Rules 20
- 1.2.7 Second Derivatives 22

1.3 Integral Calculus 24
- 1.3.1 Line, Surface, and Volume Integrals 24
- 1.3.2 The Fundamental Theorem of Calculus 29
- 1.3.3 The Fundamental Theorem for Gradients 29
- 1.3.4 The Fundamental Theorem for Divergences 31
- 1.3.5 The Fundamental Theorem for Curls 34
- 1.3.6 Integration by Parts 36

1.4 Curvilinear Coordinates 38
- 1.4.1 Spherical Coordinates 38
- 1.4.2 Cylindrical Coordinates 43

1.5 The Dirac Delta Function 45
- 1.5.1 The Divergence of $\hat{\mathbf{r}}/r^2$ 45
- 1.5.2 The One-Dimensional Dirac Delta Function 46
- 1.5.3 The Three-Dimensional Delta Function 50

1.6 The Theory of Vector Fields 52
1.6.1 The Helmholtz Theorem 52
1.6.2 Potentials 53

2 ■ Electrostatics 59

2.1 The Electric Field 59
2.1.1 Introduction 59
2.1.2 Coulomb's Law 60
2.1.3 The Electric Field 61
2.1.4 Continuous Charge Distributions 63
2.2 Divergence and Curl of Electrostatic Fields 66
2.2.1 Field Lines, Flux, and Gauss's Law 66
2.2.2 The Divergence of E 71
2.2.3 Applications of Gauss's Law 71
2.2.4 The Curl of E 77
2.3 Electric Potential 78
2.3.1 Introduction to Potential 78
2.3.2 Comments on Potential 80
2.3.3 Poisson's Equation and Laplace's Equation 83
2.3.4 The Potential of a Localized Charge Distribution 84
2.3.5 Boundary Conditions 88
2.4 Work and Energy in Electrostatics 91
2.4.1 The Work It Takes to Move a Charge 91
2.4.2 The Energy of a Point Charge Distribution 92
2.4.3 The Energy of a Continuous Charge Distribution 94
2.4.4 Comments on Electrostatic Energy 96
2.5 Conductors 97
2.5.1 Basic Properties 97
2.5.2 Induced Charges 99
2.5.3 Surface Charge and the Force on a Conductor 103
2.5.4 Capacitors 105

3 ■ Potentials 113

3.1 Laplace's Equation 113
3.1.1 Introduction 113
3.1.2 Laplace's Equation in One Dimension 114
3.1.3 Laplace's Equation in Two Dimensions 115
3.1.4 Laplace's Equation in Three Dimensions 117
3.1.5 Boundary Conditions and Uniqueness Theorems 119
3.1.6 Conductors and the Second Uniqueness Theorem 121

3.2 The Method of Images 124
3.2.1 The Classic Image Problem 124
3.2.2 Induced Surface Charge 125
3.2.3 Force and Energy 126
3.2.4 Other Image Problems 127
3.3 Separation of Variables 130
3.3.1 Cartesian Coordinates 131
3.3.2 Spherical Coordinates 141
3.4 Multipole Expansion 151
3.4.1 Approximate Potentials at Large Distances 151
3.4.2 The Monopole and Dipole Terms 154
3.4.3 Origin of Coordinates in Multipole Expansions 157
3.4.4 The Electric Field of a Dipole 158

4 ■ Electric Fields in Matter 167

4.1 Polarization 167
4.1.1 Dielectrics 167
4.1.2 Induced Dipoles 167
4.1.3 Alignment of Polar Molecules 170
4.1.4 Polarization 172
4.2 The Field of a Polarized Object 173
4.2.1 Bound Charges 173
4.2.2 Physical Interpretation of Bound Charges 176
4.2.3 The Field Inside a Dielectric 179
4.3 The Electric Displacement 181
4.3.1 Gauss's Law in the Presence of Dielectrics 181
4.3.2 A Deceptive Parallel 184
4.3.3 Boundary Conditions 185
4.4 Linear Dielectrics 185
4.4.1 Susceptibility, Permittivity, Dielectric Constant 185
4.4.2 Boundary Value Problems with Linear Dielectrics 192
4.4.3 Energy in Dielectric Systems 197
4.4.4 Forces on Dielectrics 202

5 ■ Magnetostatics 210

5.1 The Lorentz Force Law 210
5.1.1 Magnetic Fields 210
5.1.2 Magnetic Forces 212
5.1.3 Currents 216
5.2 The Biot-Savart Law 223
5.2.1 Steady Currents 223
5.2.2 The Magnetic Field of a Steady Current 224

5.3 The Divergence and Curl of B 229
5.3.1 Straight-Line Currents 229
5.3.2 The Divergence and Curl of B 231
5.3.3 Ampère's Law 233
5.3.4 Comparison of Magnetostatics and Electrostatics 241
5.4 Magnetic Vector Potential 243
5.4.1 The Vector Potential 243
5.4.2 Boundary Conditions 249
5.4.3 Multipole Expansion of the Vector Potential 252

6 ■ Magnetic Fields in Matter 266

6.1 Magnetization 266
6.1.1 Diamagnets, Paramagnets, Ferromagnets 266
6.1.2 Torques and Forces on Magnetic Dipoles 266
6.1.3 Effect of a Magnetic Field on Atomic Orbits 271
6.1.4 Magnetization 273
6.2 The Field of a Magnetized Object 274
6.2.1 Bound Currents 274
6.2.2 Physical Interpretation of Bound Currents 277
6.2.3 The Magnetic Field Inside Matter 279
6.3 The Auxiliary Field H 279
6.3.1 Ampère's Law in Magnetized Materials 279
6.3.2 A Deceptive Parallel 283
6.3.3 Boundary Conditions 284
6.4 Linear and Nonlinear Media 284
6.4.1 Magnetic Susceptibility and Permeability 284
6.4.2 Ferromagnetism 288

7 ■ Electrodynamics 296

7.1 Electromotive Force 296
7.1.1 Ohm's Law 296
7.1.2 Electromotive Force 303
7.1.3 Motional emf 305
7.2 Electromagnetic Induction 312
7.2.1 Faraday's Law 312
7.2.2 The Induced Electric Field 317
7.2.3 Inductance 321
7.2.4 Energy in Magnetic Fields 328
7.3 Maxwell's Equations 332
7.3.1 Electrodynamics Before Maxwell 332
7.3.2 How Maxwell Fixed Ampère's Law 334
7.3.3 Maxwell's Equations 337

7.3.4 Magnetic Charge 338
7.3.5 Maxwell's Equations in Matter 340
7.3.6 Boundary Conditions 342

8 ■ Conservation Laws 356

8.1 Charge and Energy 356
8.1.1 The Continuity Equation 356
8.1.2 Poynting's Theorem 357
8.2 Momentum 360
8.2.1 Newton's Third Law in Electrodynamics 360
8.2.2 Maxwell's Stress Tensor 362
8.2.3 Conservation of Momentum 366
8.2.4 Angular Momentum 370
8.3 Magnetic Forces Do No Work 373

9 ■ Electromagnetic Waves 382

9.1 Waves in One Dimension 382
9.1.1 The Wave Equation 382
9.1.2 Sinusoidal Waves 385
9.1.3 Boundary Conditions: Reflection and Transmission 388
9.1.4 Polarization 391
9.2 Electromagnetic Waves in Vacuum 393
9.2.1 The Wave Equation for E and B 393
9.2.2 Monochromatic Plane Waves 394
9.2.3 Energy and Momentum in Electromagnetic Waves 398
9.3 Electromagnetic Waves in Matter 401
9.3.1 Propagation in Linear Media 401
9.3.2 Reflection and Transmission at Normal Incidence 403
9.3.3 Reflection and Transmission at Oblique Incidence 405
9.4 Absorption and Dispersion 412
9.4.1 Electromagnetic Waves in Conductors 412
9.4.2 Reflection at a Conducting Surface 416
9.4.3 The Frequency Dependence of Permittivity 417
9.5 Guided Waves 425
9.5.1 Wave Guides 425
9.5.2 TE Waves in a Rectangular Wave Guide 428
9.5.3 The Coaxial Transmission Line 431

10 ■ Potentials and Fields 436

10.1 The Potential Formulation 436
10.1.1 Scalar and Vector Potentials 436
10.1.2 Gauge Transformations 439

10.1.3 Coulomb Gauge and Lorenz Gauge 440
10.1.4 Lorentz Force Law in Potential Form 442
10.2 Continuous Distributions 444
10.2.1 Retarded Potentials 444
10.2.2 Jefimenko's Equations 449
10.3 Point Charges 451
10.3.1 Liénard-Wiechert Potentials 451
10.3.2 The Fields of a Moving Point Charge 456

11 ■ Radiation 466
11.1 Dipole Radiation 466
11.1.1 What is Radiation? 466
11.1.2 Electric Dipole Radiation 467
11.1.3 Magnetic Dipole Radiation 473
11.1.4 Radiation from an Arbitrary Source 477
11.2 Point Charges 482
11.2.1 Power Radiated by a Point Charge 482
11.2.2 Radiation Reaction 488
11.2.3 The Mechanism Responsible for the Radiation Reaction 492

12 ■ Electrodynamics and Relativity 502
12.1 The Special Theory of Relativity 502
12.1.1 Einstein's Postulates 502
12.1.2 The Geometry of Relativity 508
12.1.3 The Lorentz Transformations 519
12.1.4 The Structure of Spacetime 525
12.2 Relativistic Mechanics 532
12.2.1 Proper Time and Proper Velocity 532
12.2.2 Relativistic Energy and Momentum 535
12.2.3 Relativistic Kinematics 537
12.2.4 Relativistic Dynamics 542
12.3 Relativistic Electrodynamics 550
12.3.1 Magnetism as a Relativistic Phenomenon 550
12.3.2 How the Fields Transform 553
12.3.3 The Field Tensor 562
12.3.4 Electrodynamics in Tensor Notation 565
12.3.5 Relativistic Potentials 569

A ■ Vector Calculus in Curvilinear Coordinates 575
A.1 Introduction 575
A.2 Notation 575

A.3 Gradient 576
A.4 Divergence 577
A.5 Curl 579
A.6 Laplacian 581

B ■ The Helmholtz Theorem **582**

C ■ Units **585**

Index **589**