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B ■ The Helmholtz Theorem

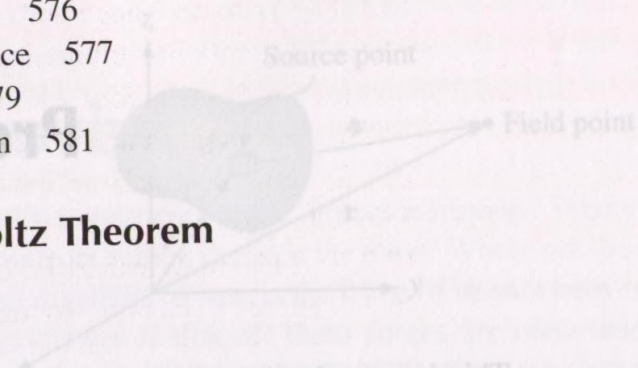
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This is a textbook on electricity and magnetism, designed for an undergraduate course at the junior or senior level. It covers topics (AC circuits, antennas, etc.) that are not usually covered in two semesters. Some have a more practical approach, while others are more theoretical. The book is divided into two parts: Part I covers electrostatics and magnetostatics, and Part II covers electrodynamics. The book is written in a clear, concise style, and includes many examples and problems. The author, Edward Purcell, is a well-known physicist and educator. The book is widely used in physics courses and is highly recommended.

For this new edition I have made a large number of small changes, in the interest of clarity and grace. In a few places I have corrected serious errors, added some problems and examples (and removed a few that were not effective). And I have included more references to the accessible literature (particularly the American Journal of Physics). I realize, of course, that most readers will not have the time or inclination to consult these resources, but I think it is worthwhile anyway, if only to emphasize that electrodynamics, notwithstanding its venerable age, is very much alive and maintaining new discoveries are being made all the time. I hope that occasionally a problem will pique your curiosity, and you will be prompted to look to the references, some of them are real gems. I have maintained three tiers of notation: the first is the standard notation used in most textbooks (Lewis and Herschel Snodgrass (Lewis and Snodgrass), the second is the notation used in the American Journal of Physics (AJP), and the third is the notation used in the American Journal of Physics (AJP). In general, all units are in SI units, but in some places I have used Gaussian units for convenience. The distance from the z axis in cylindrical coordinates is designated by ρ , to avoid confusion with r (the distance from the origin, and the radial coordinate in spherical coordinates).

- The script letter \mathbf{r} denotes the vector from a source point \mathbf{r}' to the field point \mathbf{r} (see Figure). Some authors prefer the more explicit $(\mathbf{r} - \mathbf{r}')$. But this makes many equations distressingly cumbersome, especially when the unit vector $\hat{\mathbf{z}}$ is involved. I realize that many readers are tempted to interpret \mathbf{r} as r —it certainly makes the integrals easier! Please take note: $\mathbf{r} \equiv (\mathbf{r} - \mathbf{r}')$, which is not the same as r . I think it's good notation, but it does have to be handled with care.

In MS Word, \mathbf{r} is "Kaufmann font", but this is very difficult to install in TeX. TeX use can download a pretty good facsimile from my web site.