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Note that, if G is a right partially ordered group, then the order is completely determined by $G^+ = \{g \in G : g \geq 1\}$, since $f \leq g$ if and only if $1 \leq gf^{-1}$. We call G^+ the set of positive elements of G . Similarly, if G is a left partially ordered group, then the order is completely determined by G^+ .