

Contents

Exponential Families	141
5 Density Estimation	141
5.1 Introduction	141
5.2 Poly Tree Priors	141
5.3 The Space $M(X)$	141
5.4 (Prior) Probability Measures on $M(X)$	141
5.5 Finite	141
5.5.1 $X = \mathbb{R}$	141
5.5.2 $X = \mathbb{R}$	141
5.5.3 Tail Free Priors	141
5.5.4 Tail Free Priors and 0-1-Laws	141
5.5.5 Space of Probability Measures on $M(\mathbb{R})$	141
5.5.6 De Finetti's Theorem	141
5.5.7 Extensions	141
5.6 Dirichlet and Poly tree process	141
5.7 Dirichlet and Poly tree process	141
5.8 Finite Dimensional Dirichlet Distribution	141
5.9 Dirichlet Distribution via Poly Tree Process	141
Introduction: Why Bayesian Nonparametrics—An Overview and Summary	1
1 Preliminaries and the Finite Dimensional Case	9
1.1 Introduction	9
1.2 Metric Spaces	10
1.2.1 preliminaries	10
1.2.2 Weak Convergence	12
1.3 Posterior Distribution and Consistency	15
1.3.1 Preliminaries	15
1.3.2 Posterior Consistency and Posterior Robustness	18
1.3.3 Doob's Theorem	22
1.3.4 Wald-Type Conditions	24
1.4 Asymptotic Normality of MLE and Bernstein–von Mises Theorem	33
1.5 Ibragimov and Hasminskii Conditions	41
1.6 Nonsubjective Priors	46
1.6.1 Fully Specified	46
1.6.2 Discussion	52
1.7 Conjugate and Hierarchical Priors	52

1.8	Exchangeability, De Finetti's Theorem, Exponential Families	54
2	$M(\mathcal{X})$ and Priors on $M(\mathcal{X})$	57
2.1	Introduction	57
2.2	The Space $M(\mathcal{X})$	58
2.3	(Prior) Probability Measures on $M(\mathcal{X})$	62
2.3.1	\mathcal{X} Finite	62
2.3.2	$\mathcal{X} = \mathbb{R}$	64
2.3.3	Tail Free Priors	70
2.4	Tail Free Priors and 0-1 Laws	75
2.5	Space of Probability Measures on $M(\mathbb{R})$	78
2.6	De Finetti's Theorem	83
3	Dirichlet and Polya tree process	87
3.1	Dirichlet and Polya tree process	87
3.1.1	Finite Dimensional Dirichlet Distribution	87
3.1.2	Dirichlet Distribution via Polya Urn Scheme	94
3.2	Dirichlet Process on $M(\mathbb{R})$	96
3.2.1	Construction and Properties	96
3.2.2	The Sethuraman Construction	103
3.2.3	Support of D_α	104
3.2.4	Convergence Properties of D_α	105
3.2.5	Elicitation and Some Applications	107
3.2.6	Mutual Singularity of Dirichlet Priors	110
3.2.7	Mixtures of Dirichlet Process	113
3.3	Polya Tree Process	114
3.3.1	The Finite Case	114
3.3.2	$\mathcal{X} = \mathbb{R}$	116
4	Consistency Theorems	121
4.1	Introduction	121
4.2	Preliminaries	122
4.3	Finite and Tail free case	124
4.4	Posterior Consistency on Densities	126
4.4.1	Schwartz Theorem	126
4.4.2	L_1 -Consistency	132

4.5	Consistency via LeCam's inequality	137
5	Density Estimation	141
5.1	Introduction	141
5.2	Polya Tree Priors	142
5.3	Mixtures of Kernels	143
5.4	Hierarchical Mixtures	147
5.5	Random Histograms	148
5.5.1	Weak Consistency	150
5.5.2	L_1 -Consistency	156
5.6	Mixtures of Normal Kernel	161
5.6.1	Dirichlet Mixtures: Weak Consistency	161
5.6.2	Dirichlet Mixtures: L_1 -Consistency	169
5.6.3	Extensions	172
5.7	Gaussian Process Priors	174
6	Inference for Location Parameter	181
6.1	Introduction	181
6.2	The Diaconis-Freedman Example	182
6.3	Consistency of the Posterior	185
6.4	Polya Tree Priors	189
7	Regression Problems	197
7.1	Introduction	197
7.2	Schwartz Theorem	198
7.3	Exponentially Consistent Tests	201
7.4	Prior Positivity of Neighborhoods	206
7.5	Polya Tree Priors	208
7.6	Dirichlet Mixture of Normals	209
7.7	Binary Response Regression with Unknown Link	212
7.8	Stochastic Regressor	215
7.9	Simulations	215
8	Uniform Distribution on Infinite-Dimensional Spaces	221
8.1	Introduction	221
8.2	Towards a Uniform Distribution	222
8.2.1	The Jeffreys Prior	222
8.2.2	Uniform Distribution via Sieves and Packing Numbers	223

8.3	Technical Preliminaries	224
8.4	The Jeffreys Prior Revisited	225
8.5	Posterior Consistency for Noninformative Priors for Infinite-Dimensional Problems	229
8.6	Convergence of Posterior at Optimal Rate	231
9	Survival Analysis—Dirichlet Priors	237
9.1	Introduction	237
9.2	Dirichlet Prior	238
9.3	Cumulative Hazard Function, Identifiability	242
9.4	Priors via Distributions of (Z, δ)	247
9.5	Interval Censored Data	249
10	Neutral to the Right Priors	253
10.1	Introduction	253
10.2	Neutral to the Right Priors	254
10.3	Independent Increment Processes	258
10.4	Basic Properties	262
10.5	Beta Processes	265
	10.5.1 Definition and Construction	265
	10.5.2 Properties	268
10.6	Posterior Consistency	271
11	Exercises	281
	References	285
	Index	300