

Contents

1 LINEAR EQUATIONS	3
1.1 Introduction to linear equations	3
1.2 Solving linear equations	4
1.3 The Gauss–Jordan algorithm	5
1.4 Systematic solution of linear systems	6
1.5 Homogeneous systems	8
2 MATRICES	9
2.1 Matrix arithmetic	9
2.2 Linear transformations	11
2.3 Non-singular matrices	13
2.4 Least squares solution of equations	18
3 SUBSPACES	20
3.1 Introduction	20
3.2 Subspaces of vectors	20
3.3 Linear dependence	22
3.4 Basis of a subspace	25
3.5 Rank and nullity of a matrix	27
References	31

The matrix

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

is called the *coefficient matrix* of the system, while the matrix

$$\left[\begin{array}{c|ccccc} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array} \right]$$

is called the *augmented matrix* of the system.

Geometrically, solving a system of linear equations in two or three unknowns is equivalent to determining whether or not a family of lines (or planes) share a common point of intersection.