

Contents

Introduction	11
Preface	14
0 Preliminaries	15
0.1 Basic Mathematical Notation	15
0.2 Methods of Mathematical Proof	17
0.3 Powers, Exponents, Logs and Complex Numbers	19
1 Linear Algebra	23
1.1 Matrix Algebra	23
1.2 Systems of Linear Equations	33
1.3 Quadratic Forms	38
1.4 Eigenvalues and Eigenvectors	40
1.5 Diagonalization and Spectral Theorems	41
1.6 Appendix: Vector Spaces	47
2 Calculus	55
2.1 The Concept of Limit	55
2.2 Differentiation – the Case of One Variable	57
2.3 Rules of Differentiation	61
2.4 Maxima and Minima of a Function of One Variable	63
2.5 Integration	68
2.6 Functions of More than One Variable	75
2.7 Multivariate Unconstrained Optimization	78
2.8 The Implicit Function Theorem	81
2.9 (Quasi)Concavity and (Quasi)Convexity	87
2.10 Appendix: Matrix Derivatives	90
2.11 Appendix: Topological Structure and Its Implications	94
2.12 Appendix: Correspondences and Fixed-Point Theorems .	97

3 Constrained Optimization	101
3.1 Optimization with Equality Constraints	101
3.2 The Case of Inequality Constraints	110
3.2.1 Non-Linear Programming	111
3.2.2 Kuhn-Tucker Conditions	113
3.3 Appendix: Linear Programming	122
4 Dynamics	135
4.1 Differential Equations	135
4.1.1 Differential Equations of the First Order	135
4.1.2 Qualitative Theory of First-Order Differential Equations	140
4.1.3 Linear Differential Equations of a Higher Order with Constant Coefficients	141
4.1.4 Systems of First-Order Linear Differential Equations	145
4.1.5 Simultaneous Differential Equations and Types of Equilibria	157
4.2 Difference Equations	161
4.2.1 First-Order Linear Difference Equations	163
4.2.2 Second-Order Linear Difference Equations	167
4.2.3 The General Case of Order n	169
4.2.4 Systems of Simultaneous First-Order Difference Equations with Constant Coefficients	170
4.3 Introduction to Dynamic Optimization	172
4.3.1 First-Order Conditions	172
4.3.2 Present-Value and Current-Value Hamiltonians .	176
4.3.3 Dynamic Problems with Inequality Constraints	177
5 Exercises	185
5.1 Practice Problems	186
5.1.1 Problems	186
5.1.2 Answers	198
5.2 Solved Problems	205
5.2.1 Linear Algebra	205
5.2.2 Calculus	213
5.2.3 Constrained Optimization	234
5.2.4 Dynamics	248

5.3	Economics Applications	266
5.4	Written Assignments	276
5.5	Sample Problem Sets	309
5.6	Unsolved Problems	354
5.6.1	More Problems	354
5.6.2	Sample Tests	359