

# Table of Contents

<b>Chapter 1</b>	<b>Introduction to the Analysis of Covariance</b> .....	1
1.1	Introduction.....	1
1.2	The Covariate Adjustment Process .....	1
1.3	A General AOC Model and the Basic Philosophy .....	7
	References.....	10
<b>Chapter 2</b>	<b>One-Way Analysis of Covariance — One Covariate in a Completely Randomized Design Structure</b> .....	11
2.1	The Model .....	11
2.2	Estimation .....	12
2.3	Strategy for Determining the Form of the Model .....	14
2.4	Comparing the Treatments or Regression Lines .....	17
2.4.1	Equal Slopes Model .....	18
2.4.2	Unequal Slopes Model-Covariate by Treatment Interaction .....	21
2.5	Confidence Bands about the Difference of Two Treatments.....	25
2.6	Summary of Strategies .....	25
2.7	Analysis of Covariance Computations via the SAS® System .....	26
2.7.1	Using PROC GLM and PROC MIXED .....	26
2.7.2	Using JMP® .....	31
2.8	Conclusions.....	38
	References.....	39
	Exercise.....	39
<b>Chapter 3</b>	<b>Examples: One-Way Analysis of Covariance — One Covariate in a Completely Randomized Design Structure</b> .....	41
3.1	Introduction.....	41
3.2	Chocolate Candy — Equal Slopes.....	41
3.2.1	Analysis Using PROC GLM.....	42
3.2.2	Analysis Using PROC MIXED.....	47
3.2.3	Analysis Using JMP® .....	50
3.3	Exercise Programs and Initial Resting Heart Rate — Unequal Slopes .....	54
3.4	Effect of Diet on Cholesterol Level: An Exception to the Basic Analysis of Covariance Strategy .....	66
3.5	Change from Base Line Analysis Using Effect of Diet on Cholesterol Level Data.....	70
3.6	Shoe Tread Design Data for Exception to the Basic Strategy .....	74



3.7	Equal Slopes within Groups of Treatments and Unequal Slopes between Groups .....	78
3.8	Unequal Slopes and Equal Intercepts — Part 1 .....	83
3.9	Unequal Slopes and Equal Intercepts — Part 2 .....	85
	References .....	90
	Exercises .....	90

**Chapter 4** Multiple Covariates in a One-Way Treatment Structure in a Completely Randomized Design Structure..... 93

4.1	Introduction .....	93
4.2	The Model .....	93
4.3	Estimation.....	95
4.4	Example: Driving A Golf Ball with Different Shafts .....	95
4.5	Example: Effect of Herbicides on the Yield of Soybeans — Three Covariates .....	99
4.6	Example: Models That Are Quadratic Functions of the Covariate .....	105
4.7	Example: Comparing Response Surface Models .....	112
	Reference .....	121
	Exercises .....	121

**Chapter 5** Two-Way Treatment Structure and Analysis of Covariance in a Completely Randomized Design Structure..... 123

5.1	Introduction.....	123
5.2	The Model .....	123
5.3	Using the SAS® System .....	127
	5.3.1 Using PROC GLM and PROC MIXED .....	128
	5.3.2 Using JMP® .....	129
5.4	Example: Average Daily Gains and Birth Weight — Common Slope .....	130
5.5	Example: Energy from Wood of Different Types of Trees — Some Unequal Slopes.....	136
5.6	Missing Treatment Combinations .....	144
5.7	Example: Two-Way Treatment Structure with Missing Cells .....	147
5.8	Extensions.....	158
	Reference .....	160
	Exercises .....	160

**Chapter 6** Beta-Hat Models..... 163

6.1	Introduction.....	163
6.2	The Beta-Hat Model and Analysis.....	163
6.3	Testing Equality of Parameters .....	165
6.4	Complex Treatment Structures.....	166
6.5	Example: One-Way Treatment Structure .....	167
6.6	Example: Two-Way Treatment Structure.....	171



6.7	Summary .....	174
	Exercises .....	174
<b>Chapter 7</b>	<b>Variable Selection in the Analysis of Covariance Model .....</b>	<b>175</b>
7.1	Introduction.....	175
7.2	Procedure for Equal Slopes.....	175
7.3	Example: One-Way Treatment Structure with Equal Slopes Model.....	177
7.4	Some Theory .....	184
7.5	When Slopes are Possibly Unequal .....	185
	References.....	186
	Exercises .....	186
<b>Chapter 8</b>	<b>Comparing Models for Several Treatments .....</b>	<b>189</b>
8.1	Introduction.....	189
8.2	Testing Equality of Models for a One-Way Treatment Structure .....	190
8.3	Comparing Models for a Two-Way Treatment Structure .....	191
8.4	Example: One-Way Treatment Structure with One Covariate .....	193
8.5	Example: One-Way Treatment Structure with Three Covariates .....	195
8.6	Example: Two-Way Treatment Structure with One Covariate .....	197
8.7	Discussion.....	200
	References.....	201
	Exercises .....	201
<b>Chapter 9</b>	<b>Two Treatments in a Randomized Complete Block Design Structure.....</b>	<b>203</b>
9.1	Introduction.....	203
9.2	Complete Block Designs.....	203
9.3	Within Block Analysis.....	204
9.4	Between Block Analysis .....	206
9.5	Combining Within Block and Between Block Information .....	207
9.6	Determining the Form of the Model.....	209
9.7	Common Slope Model .....	211
9.8	Comparing the Treatments .....	214
	9.8.1 Equal Slopes Models.....	215
	9.8.2 Unequal Slopes Model .....	215
9.9	Confidence Intervals about Differences of Two Regression Lines .....	215
	9.9.1 Within Block Analysis.....	216
	9.9.2 Combined Within Block and Between Block Analysis.....	216
9.10	Computations for Model 9.1 Using the SAS® System.....	217
9.11	Example: Effect of Drugs on Heart Rate .....	221
9.12	Summary.....	226
	References.....	231
	Exercises .....	231



<b>Chapter 10</b>	<b>More Than Two Treatments in a Blocked Design Structure</b>	233
10.1	Introduction	233
10.2	RCB Design Structure — Within and Between Block Information	233
10.3	Incomplete Block Design Structure — Within and Between Block Information	234
10.4	Combining Between Block and Within Block Information	236
10.5	Example: Five Treatments in RCB Design Structure	240
10.6	Example: Balanced Incomplete Block Design Structure with Four Treatments	247
10.7	Example: Balanced Incomplete Block Design Structure with Four Treatments Using JMP®	251
10.8	Summary	254
	References	256
	Exercises	256
<b>Chapter 11</b>	<b>Covariate Measured on the Block in RCB and Incomplete Block Design Structures</b>	259
11.1	Introduction	259
11.2	The Within Block Model	260
11.3	The Between Block Model	261
11.4	Combining Within Block and Between Block Information	261
11.5	Common Slope Model	263
11.6	Adjusted Means and Comparing Treatments	264
	11.6.1 Common Slope Model	264
	11.6.2 Non-Parallel Lines Model	264
11.7	Example: Two Treatments	265
11.8	Example: Four Treatments in RCB	269
11.9	Example: Four Treatments in BIB	277
11.10	Summary	282
	References	284
	Exercises	284
<b>Chapter 12</b>	<b>Random Effects Models with Covariates</b>	287
12.1	Introduction	287
12.2	The Model	287
12.3	Estimation of the Variance Components	292
12.4	Changing Location of the Covariate Changes the Estimates of the Variance Components	297
12.5	Example: Balanced One-Way Treatment Structure	299
12.6	Example: Unbalanced One-Way Treatment Structure	304
12.7	Example: Two-Way Treatment Structure	309
12.8	Summary	315
	References	320
	Exercises	321



<b>Chapter 13</b>	<b>Mixed Models</b> .....	325
13.1	Introduction.....	325
13.2	The Matrix Form of the Mixed Model.....	325
13.3	Fixed Effects Treatment Structure.....	329
13.4	Estimation of Fixed Effects and Some Small Sample Size Approximations.....	329
13.5	Fixed Treatments and Locations Random.....	331
13.6	Example: Two-Way Mixed Effects Treatment Structure in a CRD.....	332
13.7	Example: Treatments are Fixed and Locations are Random with a RCB at Each Location.....	337
	References.....	350
	Exercises.....	351
<b>Chapter 14</b>	<b>Analysis of Covariance Models with Heterogeneous Errors</b> .....	353
14.1	Introduction.....	353
14.2	The Unequal Variance Model.....	353
14.3	Tests for Homogeneity of Variances.....	354
14.3.1	Levene's Test for Equal Variances.....	354
14.3.2	Hartley's F-Max Test for Equal Variances.....	355
14.3.3	Bartlett's Test for Equal Variances.....	355
14.3.4	Likelihood Ratio Test for Equal Variances.....	356
14.4	Estimating the Parameters of the Regression Model.....	356
14.4.1	Least Squares Estimation.....	356
14.4.2	Maximum Likelihood Methods.....	357
14.5	Determining the Form of the Model.....	357
14.6	Comparing the Models.....	359
14.6.1	Comparing the Nonparallel Lines Models.....	359
14.6.2	Comparing the Parallel Lines Models.....	361
14.7	Computational Issues.....	362
14.8	Example: One-Way Treatment Structure with Unequal Variances.....	362
14.9	Example: Two-Way Treatment Structure with Unequal Variances.....	369
14.10	Example: Treatments in Multi-location Trial.....	381
14.11	Summary.....	389
	References.....	389
	Exercises.....	389
<b>Chapter 15</b>	<b>Analysis of Covariance for Split-Plot and Strip-Plot Design Structures</b> .....	391
15.1	Introduction.....	391
15.2	Some Concepts.....	392
15.3	Covariate Measured on the Whole Plot or Large Size of Experimental Unit.....	392
15.4	Covariate is Measured on the Small Size of Experimental Unit.....	395



15.5	Covariate is Measured on the Large Size of Experimental Unit and a Covariate is Measured on the Small Size of Experimental Unit .....	398
15.6	General Representation of the Covariate Part of the Model .....	399
15.6.1	Covariate Measured on Large Size of Experimental Unit .....	401
15.6.2	Covariate Measured on the Small Size of Experimental Units .....	403
15.6.3	Summary of General Representation .....	405
15.7	Example: Flour Milling Experiment — Covariate Measured on the Whole Plot .....	406
15.8	Example: Cookie Baking .....	414
15.9	Example: Teaching Methods with One Covariate Measured on the Large Size Experimental Unit and One Covariate Measured on the Small Size Experimental Unit .....	426
15.10	Example: Comfort Study in a Strip-Plot Design with Three Sizes of Experimental Units and Three Covariates .....	432
15.11	Conclusions .....	444
	References .....	446
	Exercises .....	446

## **Chapter 16** Analysis of Covariance for Repeated Measures Designs .....

16.1	Introduction .....	451
16.2	The Covariance Part of the Model — Selecting R .....	453
16.3	Covariance Structure of the Data .....	456
16.4	Specifying the Random and Repeated Statements for PROC MIXED of the SAS® System .....	457
16.5	Selecting an Adequate Covariance Structure .....	458
16.6	Example: Systolic Blood Pressure Study with Covariate Measured on the Large Size Experimental Unit .....	459
16.7	Example: Oxide Layer Development Experiment with Three Sizes of Experimental Units Where the Repeated Measure is at the Middle Size of Experimental Unit and the Covariate is Measured on the Small Size Experimental Unit .....	470
16.8	Conclusions .....	479
	References .....	487
	Exercises .....	487

## **Chapter 17** Analysis of Covariance for Nonreplicated Experiments .....

17.1	Introduction .....	493
17.2	Experiments with A Single Covariate .....	495
17.3	Experiments with Multiple Covariates .....	499
17.4	Selecting Non-null and Null Partitions .....	501
17.5	Estimating the Parameters .....	502
17.6	Example: Milling Flour Using Three Factors Each at Two Levels .....	503
17.7	Example: Baking Bread Using Four Factors Each at Two Levels .....	508
17.8	Example: Hamburger Patties with Four Factors Each at Two Levels .....	511



17.9	Example: Strength of Composite Material Coupons with Two Covariates .....	512
17.10	Example: Effectiveness of Paint on Bricks with Unequal Slopes.....	520
17.11	Summary.....	527
	References.....	529
	Exercises .....	530

## **Chapter 18** Special Applications of Analysis of Covariance..... 533

18.1	Introduction.....	533
18.2	Blocking and Analysis of Covariance.....	533
18.3	Treatments Have Different Ranges of the Covariate.....	543
18.4	Nonparametric Analysis of Covariance .....	552
	18.4.1 Heart Rate Data from Exercise Programs .....	552
	18.4.2 Average Daily Gain Data from a Two-Way Treatment Structure.....	555
18.5	Crossover Design with Covariates .....	559
18.6	Nonlinear Analysis of Covariance .....	564
18.7	Effect of Outliers.....	572
	References.....	590
	Exercises .....	590

<b>Index</b> .....	597
--------------------	-----