Contents

Preface

page vii

V

Basic probability	1
Discrete outcomes	3
A uniform distribution	3
Conditional Probabilities. The Bayes Theorem. Independent trials	6
The exclusion-inclusion formula. The ballot problem	27
Random variables. Expectation and conditional expectation.	
Joint distributions	33
The binomial, Poisson and geometric distributions. Probability	
generating, moment generating and characteristic functions	54
Chebyshev's and Markov's inequalities. Jensen's inequality. The Law	
of Large Numbers and the De Moivre-Laplace Theorem	75
Branching processes	96
Continuous outcomes	108
Uniform distribution. Probability density functions. Random variables.	
Independence	108
Expectation, conditional expectation, variance, generating function,	
characteristic function	142
Normal distributions. Convergence of random variables	
and distributions. The Central Limit Theorem	168
Basic statistics	101
	 Basic probability Discrete outcomes A uniform distribution Conditional Probabilities. The Bayes Theorem. Independent trials The exclusion-inclusion formula. The ballot problem Random variables. Expectation and conditional expectation. Joint distributions The binomial, Poisson and geometric distributions. Probability generating, moment generating and characteristic functions Chebyshev's and Markov's inequalities. Jensen's inequality. The Law of Large Numbers and the De Moivre–Laplace Theorem Branching processes Continuous outcomes Uniform distribution. Probability density functions. Random variables. Independence Expectation, conditional expectation, variance, generating function, characteristic function Normal distributions. Convergence of random variables and distributions. The Central Limit Theorem

3	Parameter estimation	193
3.1	Preliminaries. Some important probability distributions	193
3.2	Estimators. Unbiasedness	204
3.3	Sufficient statistics. The factorisation criterion	209
3.4	Maximum likelihood estimators	213
3.5	Normal samples. The Fisher Theorem	215

	a
V1	Contents
• •	0011101110

3.6	Mean square errors. The Rao–Blackwell Theorem.	
	The Cramér–Rao inequality	218
3.7	Exponential families	225
3.8	Confidence intervals	229
3.9	Bayesian estimation	233
4	Hypothesis testing	242
4.1	Type I and type II error probabilities. Most powerful tests	242
4.2	Likelihood ratio tests. The Neyman-Pearson Lemma and beyond	243
4.3	Goodness of fit. Testing normal distributions, 1: homogeneous samples	252
4.4	The Pearson χ^2 test. The Pearson Theorem	257
4.5	Generalised likelihood ratio tests. The Wilks Theorem	261
4.6	Contingency tables	270
4.7	Testing normal distributions, 2: non-homogeneous samples	276
4.8	Linear regression. The least squares estimators	289
4.9	Linear regression for normal distributions	292
5	Cambridge University Mathematical Tripos examination questions	
	in IB Statistics (1992–1999)	298
	Appendix 1 Tables of random variables and probability distributions	346
	Appendix 2 Index of Cambridge University Mathematical Tripos examination questions in IA Probability (1992–1999)	349
	Bibliography	352
	Index	358