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The data and software code for the book's examples are available on the companion website *http://bise.wceruw.org/publications.html*.