Contents

Preface vii

1 Combinatorics 1
- 1.1 Factorials . . . 2
- 1.2 Permutations . . . 3
- 1.3 Ordered sets, repetitions allowed . . . 7
- 1.4 Ordered sets, repetitions not allowed . . . 12
- 1.5 Unordered sets, repetitions not allowed . . . 14
- 1.6 What we know so far . . . 20
- 1.7 Unordered sets, repetitions allowed . . . 21
- 1.8 Binomial coefficients . . . 29
  - 1.8.1 Coins and Pascal's triangle . . . 29
  - 1.8.2 $(a + b)^n$ and Pascal's triangle . . . 31
  - 1.8.3 Properties of Pascal's triangle . . . 33
- 1.9 Summary . . . 35
- 1.10 Exercises . . . 35
- 1.11 Problems . . . 36
- 1.12 Solutions . . . 41

2 Probability 57
- 2.1 Definition of probability . . . 57
- 2.2 The rules of probability . . . 59
  - 2.2.1 AND: The "intersection" probability, $P(A$ and $B)$ . . . 60
  - 2.2.2 OR: The "union" probability, $P(A$ or $B)$ . . . 68
  - 2.2.3 (In)dependence and (non)exclusiveness . . . 71
  - 2.2.4 Conditional probability . . . 73
- 2.3 Examples . . . 75
  - 2.3.1 The art of "not" . . . 75
  - 2.3.2 Picking seats . . . 76
  - 2.3.3 Socks in a drawer . . . 79
  - 2.3.4 Coins and dice . . . 81
  - 2.3.5 Cards . . . 83
- 2.4 Four classic problems . . . 85
  - 2.4.1 The Birthday Problem . . . 85
  - 2.4.2 The Game-Show Problem . . . 87

2.4.3 The Prosecutor's Fallacy . . . 90
2.4.4 The Boy/Girl Problem . . . 93
2.5 Bayes' theorem . . . 97
2.6 Stirling's formula . . . 106
2.7 Summary . . . 108
2.8 Exercises . . . 109
2.9 Problems . . . 109
2.10 Solutions . . . 114

3 Expectation values **133**
3.1 Expectation value . . . 133
3.2 Variance . . . 141
3.3 Standard deviation . . . 146
3.4 Standard deviation of the mean . . . 150
3.5 Sample variance . . . 155
3.6 Summary . . . 163
3.7 Exercises . . . 164
3.8 Problems . . . 165
3.9 Solutions . . . 168

4 Distributions **182**
4.1 Discrete distributions . . . 182
4.2 Continuous distributions . . . 184
4.2.1 Motivation . . . 184
4.2.2 Probability density . . . 186
4.2.3 Probability equals area . . . 189
4.3 Uniform distribution . . . 191
4.4 Bernoulli distribution . . . 192
4.5 Binomial distribution . . . 193
4.6 Exponential distribution . . . 196
4.6.1 Discrete case . . . 196
4.6.2 Rates, expectation values, and probabilities . . . 199
4.6.3 Continuous case . . . 202
4.7 Poisson distribution . . . 207
4.7.1 Discrete case . . . 207
4.7.2 Continuous case . . . 209
4.8 Gaussian distribution . . . 215
4.9 Summary . . . 221
4.10 Exercises . . . 222
4.11 Problems . . . 222
4.12 Solutions . . . 227

5 Gaussian approximations **250**
5.1 Binomial and Gaussian . . . 250
5.2 The law of large numbers . . . 256
5.3 Poisson and Gaussian . . . 260
5.4 Binomial, Poisson, and Gaussian . . . 263

5.5 The central limit theorem 264
5.6 Summary 269
5.7 Exercises 270
5.8 Problems 270
5.9 Solutions 272

6 Correlation and regression **277**
6.1 The concept of correlation 277
6.2 A model for correlation 280
6.3 The correlation coefficient, r 285
6.4 Improving the prediction for Y 291
6.5 Calculating $\rho(x,y)$ 294
6.6 The standard-deviation box 297
6.7 The regression lines 300
6.8 Two regression examples 305
6.8.1 Example 1: Retaking a test 305
6.8.2 Example 2: Comparing IQ's 310
6.9 Least-squares fitting 313
6.10 Summary 317
6.11 Exercises 318
6.12 Problems 318
6.13 Solutions 320

7 Appendices **335**
7.1 Appendix A: Subtleties about probability 335
7.2 Appendix B: Euler's number, e 339
7.2.1 Definition of e 339
7.2.2 Raising e to a power 341
7.2.3 The infinite series for e^x 344
7.2.4 The slope of e^x 346
7.3 Appendix C: Approximations to $(1+a)^n$ 346
7.4 Appendix D: The slope of e^x 350
7.4.1 First derivation 350
7.4.2 Second derivation 353
7.5 Appendix E: Important results 356
7.6 Appendix F: Glossary of notation 359