

Contents

| | |
|---|-----------|
| List of Figures | xiii |
| 1. Single Species Population Dynamics | 1 |
| 1.1 Introduction | 1 |
| 1.2 Linear and Nonlinear First Order Discrete Time Models | 2 |
| 1.2.1 The Biology of Insect Population Dynamics | 3 |
| 1.2.2 A Model for Insect Population Dynamics with Competition | 4 |
| 1.3 Differential Equation Models | 11 |
| 1.4 Evolutionary Aspects | 16 |
| 1.5 Harvesting and Fisheries | 17 |
| 1.6 Metapopulations | 21 |
| 1.7 Delay Effects | 24 |
| 1.8 Fibonacci's Rabbits | 27 |
| 1.9 Leslie Matrices: Age-structured Populations in Discrete Time .. | 30 |
| 1.10 Euler-Lotka Equations | 34 |
| 1.10.1 Discrete Time | 34 |
| 1.10.2 Continuous Time | 38 |
| 1.11 The McKendrick Approach to Age Structure | 41 |
| 1.12 Conclusions | 44 |
| 2. Population Dynamics of Interacting Species | 47 |
| 2.1 Introduction | 48 |
| 2.2 Host-parasitoid Interactions | 48 |
| 2.3 The Lotka-Volterra Prey-predator Equations | 54 |
| 2.4 Modelling the Predator Functional Response | 60 |
| 2.5 Competition | 66 |

| | | |
|-----------|--|------------|
| 2.6 | Ecosystems Modelling | 70 |
| 2.7 | Interacting Metapopulations | 74 |
| 2.7.1 | Competition | 75 |
| 2.7.2 | Predation | 77 |
| 2.7.3 | Predator-mediated Coexistence of Competitors | 78 |
| 2.7.4 | Effects of Habitat Destruction | 79 |
| 2.8 | Conclusions | 81 |
| 3. | Infectious Diseases | 83 |
| 3.1 | Introduction | 83 |
| 3.2 | The Simple Epidemic and SIS Diseases | 86 |
| 3.3 | SIR Epidemics | 90 |
| 3.4 | SIR Endemics | 96 |
| 3.4.1 | No Disease-related Death | 97 |
| 3.4.2 | Including Disease-related Death | 99 |
| 3.5 | Eradication and Control | 100 |
| 3.6 | Age-structured Populations | 103 |
| 3.6.1 | The Equations | 103 |
| 3.6.2 | Steady State | 105 |
| 3.7 | Vector-borne Diseases | 107 |
| 3.8 | Basic Model for Macroparasitic Diseases | 109 |
| 3.9 | Evolutionary Aspects | 113 |
| 3.10 | Conclusions | 115 |
| 4. | Population Genetics and Evolution | 117 |
| 4.1 | Introduction | 117 |
| 4.2 | Mendelian Genetics in Populations with Non-overlapping Generations | 119 |
| 4.3 | Selection Pressure | 123 |
| 4.4 | Selection in Some Special Cases | 127 |
| 4.4.1 | Selection for a Dominant Allele | 127 |
| 4.4.2 | Selection for a Recessive Allele | 127 |
| 4.4.3 | Selection against Dominant and Recessive Alleles | 129 |
| 4.4.4 | The Additive Case | 129 |
| 4.5 | Analytical Approach for Weak Selection | 130 |
| 4.6 | The Balance Between Selection and Mutation | 131 |
| 4.7 | Wright's Adaptive Topography | 133 |
| 4.8 | Evolution of the Genetic System | 134 |
| 4.9 | Game Theory | 136 |
| 4.10 | Replicator Dynamics | 142 |
| 4.11 | Conclusions | 145 |

| | |
|---|-----|
| 5. Biological Motion | 147 |
| 5.1 Introduction | 147 |
| 5.2 Macroscopic Theory of Motion; A Continuum Approach | 148 |
| 5.2.1 General Derivation | 148 |
| 5.2.2 Some Particular Cases | 151 |
| 5.3 Directed Motion, or Taxis | 154 |
| 5.4 Steady State Equations and Transit Times | 156 |
| 5.4.1 Steady State Equations in One Spatial Variable | 156 |
| 5.4.2 Transit Times | 157 |
| 5.4.3 Macrophages vs Bacteria | 160 |
| 5.5 Biological Invasions: A Model for Muskrat Dispersal | 160 |
| 5.6 Travelling Wave Solutions of General Reaction-diffusion Equations | 164 |
| 5.6.1 Node-saddle Orbits (the Monostable Equation) | 166 |
| 5.6.2 Saddle-saddle Orbits (the Bistable Equation) | 167 |
| 5.7 Travelling Wave Solutions of Systems of Reaction-diffusion Equations: Spatial Spread of Epidemics | 168 |
| 5.8 Conclusions | 172 |
| | |
| 6. Molecular and Cellular Biology | 175 |
| 6.1 Introduction | 175 |
| 6.2 Biochemical Kinetics | 176 |
| 6.3 Metabolic Pathways | 183 |
| 6.3.1 Activation and Inhibition | 184 |
| 6.3.2 Cooperative Phenomena | 186 |
| 6.4 Neural Modelling | 191 |
| 6.5 Immunology and AIDS | 197 |
| 6.6 Conclusions | 202 |
| | |
| 7. Pattern Formation | 205 |
| 7.1 Introduction | 205 |
| 7.2 Turing Instability | 206 |
| 7.3 Turing Bifurcations | 211 |
| 7.4 Activator-inhibitor Systems | 214 |
| 7.4.1 Conditions for Turing Instability | 214 |
| 7.4.2 Short-range Activation, Long-range Inhibition | 219 |
| 7.4.3 Do Activator-inhibitor Systems Explain Biological Pattern Formation? | 223 |
| 7.5 Bifurcations with Domain Size | 224 |
| 7.6 Incorporating Biological Movement | 229 |
| 7.7 Mechanochemical Models | 233 |
| 7.8 Conclusions | 233 |

| | |
|---|-----|
| 8. Tumour Modelling | 235 |
| 8.1 Introduction | 235 |
| 8.2 Phenomenological Models | 237 |
| 8.3 Nutrients: the Diffusion-limited Stage | 240 |
| 8.4 Moving Boundary Problems | 242 |
| 8.5 Growth Promoters and Inhibitors | 245 |
| 8.6 Vascularisation | 247 |
| 8.7 Metastasis | 248 |
| 8.8 Immune System Response | 249 |
| 8.9 Conclusions | 251 |
| Further Reading | 253 |
| A. Some Techniques for Difference Equations | 257 |
| A.1 First-order Equations | 257 |
| A.1.1 Graphical Analysis | 257 |
| A.1.2 Linearisation | 258 |
| A.2 Bifurcations and Chaos for First-order Equations | 260 |
| A.2.1 Saddle-node Bifurcations | 260 |
| A.2.2 Transcritical Bifurcations | 261 |
| A.2.3 Pitchfork Bifurcations | 262 |
| A.2.4 Period-doubling or Flip Bifurcations | 263 |
| A.3 Systems of Linear Equations: Jury Conditions | 266 |
| A.4 Systems of Nonlinear Difference Equations | 267 |
| A.4.1 Linearisation of Systems | 268 |
| A.4.2 Bifurcation for Systems | 268 |
| B. Some Techniques for Ordinary Differential Equations | 271 |
| B.1 First-order Ordinary Differential Equations | 271 |
| B.1.1 Geometric Analysis | 271 |
| B.1.2 Integration | 272 |
| B.1.3 Linearisation | 272 |
| B.2 Second-order Ordinary Differential Equations | 273 |
| B.2.1 Geometric Analysis (Phase Plane) | 273 |
| B.2.2 Linearisation | 274 |
| B.2.3 Poincaré–Bendixson Theory | 276 |
| B.3 Some Results and Techniques for m th Order Systems | 277 |
| B.3.1 Linearisation | 278 |
| B.3.2 Lyapunov Functions | 278 |
| B.3.3 Some Miscellaneous Facts | 279 |
| B.4 Bifurcation Theory for Ordinary Differential Equations | 279 |
| B.4.1 Bifurcations with Eigenvalue Zero | 279 |

B.4.2 Hopf Bifurcations 280

C. Some Techniques for Partial Differential Equations 283

C.1 First-order Partial Differential Equations and Characteristics .. 283

C.2 Some Results and Techniques for the Diffusion Equation 284

C.2.1 The Fundamental Solution 284

C.2.2 Connection with Probabilities 287

C.2.3 Other Coordinate Systems 288

C.3 Some Spectral Theory for Laplace's Equation 289

C.4 Separation of Variables in Partial Differential Equations 291

C.5 Systems of Diffusion Equations with Linear Kinetics 295

C.6 Separating the Spatial Variables from Each Other 297

D. Non-negative Matrices 299

D.1 Perron–Frobenius Theory 299

E. Hints for Exercises 301

Index 329

1.1. Parameters for the Russell equation 8

1.2. Colwebbing variant 10

1.3. Logistic population growth 14

1.7. Yield-effort curves 19

1.8. The archetypal metapopulation model 22

1.9. The archetypal metapopulation growth rate 23

1.10. Instability in a delay equation 26

1.11. Perron-Frobenius 32

1.12. A Lotka diagram 41

1.13. Archetypal survivorship curves 43

1.14. The functions $\log R_0$ and $1 - 1/R_0$ 51

1.15. Some numerical solutions of the Nicholson-Bailey host-parasitoid model 51

1.16. Negative binomial and Poisson distributions 53

1.17. Results of host-parasitoid model with negative binomial search function 53

1.18. Winter moth host-parasitoid simulations and data 54

1.19. Results of Lotka-Volterra prey-predator model 57

1.20. May's population data 58

1.21. Predator functional response curves 67

1.22. Rosenzweig-MacArthur phase planes 69

1.23. Lotka-Volterra competition phase planes 69