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The concepts of crystalline solids and symmetry are just about synonymous today, although the general sense of symmetry is much older than the idea of symmetrical arrangement of atoms in the molecules of crystalline solids. Following dictionaries, symmetry can be defined as the "beauty of form arising from balanced proportions," and to be symmetrical is to have the "correspondence in size, shape, and relative position of parts on opposite sides of a dividing line or median plane or about a center or axis."

Humans constantly deal with symmetry, often without even noticing its significance in daily life. For example, our exposure to symmetry begins every morning with a glimpse in a mirror, and it ends every night when we fall asleep in a bed with balanced proportions. Although intuitive perceptions of symmetry is familiar to everyone, it has multiple applications in science. A much more comprehensive and formal description of symmetry, when compared to that found in dictionaries or thesauri, follows.

In the first five chapters of this book, we consider basic concepts of crystallographic symmetry, which are essential to the understanding of how atoms and molecules are arranged in space, and how they form crystalline solids. Further, the detailed knowledge of crystallographic symmetry is important to appreciate both the capabilities and limitations of powder diffraction techniques when they are applied to the determination of the crystal structure of solids.

We begin with the most described notions of the three-dimensional periodicity of crystals, which are crystallographic symmetry, and consider the properties and characteristics of both finite and infinite symmetry elements, including the derivations of both point and space groups.¹ The formal, algebraic treatment of

¹ *International Union of Pure and Applied Chemistry, G. & C. Merriam Company Publ., Springfield, MA, 1962.*

Crystallographic symmetry and space groups are employed to describe relationships among parts of crystals, such as geometrical figures or shapes of natural and synthetic crystals. Point and space groups establish symmetrical relationships among parts of crystals, e.g., two-dimensional wall patterns or three-dimensional lattices of atoms or molecules in crystals. Although the division of symmetry elements on