Contents

1 **Introduction** 1
1.1 Summary 1
1.2 Finance 1
1.3 Econometrics 3
1.4 Mathematics 6
1.5 Problems and Solutions 7
References 10

Part I Time Series Modeling

2 **Basic Concepts from Probability Theory** 13
2.1 Summary 13
2.2 Random Variables 13
2.3 Joint and Conditional Distributions 22
2.4 Stochastic Processes (SP) 29
2.5 Problems and Solutions 35
References 42

3 **Autoregressive Moving Average Processes (ARMA)** 45
3.1 Summary 45
3.2 Moving Average Processes 45
3.3 Lag Polynomials and Invertibility 51
3.4 Autoregressive and Mixed Processes 56
3.5 Problems and Solutions 68
References 75

4 **Spectra of Stationary Processes** 77
4.1 Summary 77
4.2 Definition and Interpretation 77
4.3 Filtered Processes 84
4.4 Examples of ARMA Spectra 89
4.5 Problems and Solutions 95
References 101

5 Long Memory and Fractional Integration 103
5.1 Summary 103
5.2 Persistence and Long Memory 103
5.3 Fractionally Integrated Noise 108
5.4 Generalizations 113
5.5 Problems and Solutions 118
References 125

6 Processes with Autoregressive Conditional Heteroskedasticity (ARCH) 127
6.1 Summary 127
6.2 Time-Dependent Heteroskedasticity 127
6.3 ARCH Models 130
6.4 Generalizations 135
6.5 Problems and Solutions 142
References 148

Part II Stochastic Integrals

7 Wiener Processes (WP) 151
7.1 Summary 151
7.2 From Random Walk to Wiener Process 151
7.3 Properties 157
7.4 Functions of Wiener Processes 161
7.5 Problems and Solutions 170
References 177

8 Riemann Integrals 179
8.1 Summary 179
8.2 Definition and Fubini's Theorem 179
8.3 Riemann Integration of Wiener Processes 183
8.4 Convergence in Mean Square 186
8.5 Problems and Solutions 190
References 197

9 Stieltjes Integrals 199
9.1 Summary 199
9.2 Definition and Partial Integration 199
9.3 Gaussian Distribution and Autocovariances 202
9.4 Standard Ornstein-Uhlenbeck Process 204
9.5 Problems and Solutions 207
Reference 211

10 Ito Integrals 213
10.1 Summary 213
10.2 A Special Case 213
10.3 General Ito Integrals 218

10.4 (Quadratic) Variation 222
10.5 Problems and Solutions 229
References 237

11 Ito's Lemma 239
11.1 Summary 239
11.2 The Univariate Case 239
11.3 Bivariate Diffusions with One WP 245
11.4 Generalization for Independent WP 250
11.5 Problems and Solutions 254
Reference 258

Part III Applications

12 Stochastic Differential Equations (SDE) 261
12.1 Summary 261
12.2 Definition and Existence 261
12.3 Linear Stochastic Differential Equations 265
12.4 Numerical Solutions 272
12.5 Problems and Solutions 273
References 282

13 Interest Rate Models 285
13.1 Summary 285
13.2 Ornstein-Uhlenbeck Process (OUP) 285
13.3 Positive Linear Interest Rate Models 288
13.4 Nonlinear Models 292
13.5 Problems and Solutions 296
References 302

14 Asymptotics of Integrated Processes 303
14.1 Summary 303
14.2 Limiting Distributions of Integrated Processes 303
14.3 Weak Convergence of Functions 310
14.4 Multivariate Limit Theory 317
14.5 Problems and Solutions 321
References 329

15 Trends, Integration Tests and Nonsense Regressions 331
15.1 Summary 331
15.2 Trend Regressions 331
15.3 Integration Tests 336
15.4 Nonsense Regression 341
15.5 Problems and Solutions 344
References 352

16 Cointegration Analysis 353
16.1 Summary 353
16.2 Error-Correction and Cointegration 353
16.3 Cointegration Regressions 358
16.4 Cointegration Testing 365
16.5 Problems and Solutions 373
References 381

References 383

Index 389