

Markov chains are central to the understanding of random processes. This is not only because they pervade applications, but also because one can calculate explicitly many quantities of interest. This textbook, aimed at advanced undergraduate or MSc students with some background in basic probability theory, focusses on Markov chains and develops quickly a coherent and rigorous theory. In a non-technical way, it explains methods of calculation for transition probabilities, hitting probabilities, long-run averages and equilibrium probabilities.

The author presents both discrete-time and continuous-time chains and also discusses reversibility. He uses random walks as important examples, as well as Poisson processes and birth-and-death processes. A distinguishing feature of the book is an introduction to more advanced topics such as martingales and potentials, in the established context of Markov chains. There are applications to simulation, economics, optimal control, genetics, queues and many other topics.

There is a careful selection of exercises and examples drawn both from theory and practice. The book will therefore be an ideal text either for elementary courses on random processes or those that are more oriented towards applications.

## **Cambridge Series in Statistical and Probabilistic Mathematics**

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This series of high quality upper-division textbooks and expository monographs covers all aspects of stochastic applicable mathematics. The topics range from pure and applied statistics to probability theory, operations research, optimization and mathematical programming. The books contain clear presentations of new developments in the field and also of the state of the art in classical methods. While emphasizing rigorous treatment of theoretical methods, the books also contain applications and discussions of new techniques made possible by advances in computational practice.

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