
Contents

Introduction	xi
Acknowledgments	xv
1 General Facts About Groups	1
1 Review of Definitions	1
2 Examples of Finite Groups	2
2.1 Cyclic Group of Order n	2
2.2 Symmetric Group \mathfrak{S}_n	3
2.3 Dihedral Group	3
2.4 Other Examples	3
3 Examples of Infinite Groups	3
4 Group Actions and Conjugacy Classes	5
References	6
Exercises	6
2 Representations of Finite Groups	9
1 Representations	9
1.1 General Facts	9
1.2 Irreducible Representations	11
1.3 Direct Sum of Representations	11
1.4 Intertwining Operators and Schur's Lemma	12
2 Characters and Orthogonality Relations	14
2.1 Functions on a Group, Matrix Coefficients	14
2.2 Characters of Representations and Orthogonality Relations	15
2.3 Character Table	18
2.4 Application to the Decomposition of Representations ..	19
3 The Regular Representation	20
3.1 Definition	20
3.2 Character of the Regular Representation	21
3.3 Isotypic Decomposition	21
3.4 Basis of the Vector Space of Class Functions	22

4	Projection Operators	24
5	Induced Representations	25
5.1	Definition	25
5.2	Geometric Interpretation	26
	References	26
	Exercises	27
3	Representations of Compact Groups	33
1	Compact Groups	33
2	Haar Measure	34
3	Representations of Topological Groups and Schur's Lemma ...	36
3.1	General Facts	36
3.2	Coefficients of a Representation	36
3.3	Intertwining Operators	37
3.4	Operations on Representations	38
3.5	Schur's Lemma	38
4	Representations of Compact Groups	39
4.1	Complete Reducibility	39
4.2	Orthogonality Relations	40
5	Summary of Chapter 3	42
	References	43
	Exercises	44
4	Lie Groups and Lie Algebras	47
1	Lie Algebras	47
1.1	Definition and Examples	47
1.2	Morphisms	49
1.3	Commutation Relations and Structure Constants	49
1.4	Real Forms	49
1.5	Representations of Lie Algebras	50
2	Review of the Exponential Map	51
3	One-Parameter Subgroups of $GL(n, \mathbb{K})$	54
4	Lie Groups	56
5	The Lie Algebra of a Lie Group	56
6	The Connected Component of the Identity	59
7	Morphisms of Lie Groups and of Lie Algebras	60
7.1	Differential of a Lie Group Morphism	60
7.2	Differential of a Lie Group Representation	62
7.3	The Adjoint Representation	64
	References	65
	Exercises	66
5	Lie Groups $SU(2)$ and $SO(3)$	71
1	The Lie Algebras $\mathfrak{su}(2)$ and $\mathfrak{so}(3)$	71
1.1	Bases of $\mathfrak{su}(2)$	71
1.2	Bases of $\mathfrak{so}(3)$	73
1.3	Bases of $\mathfrak{sl}(2, \mathbb{C})$	74

2	The Covering Morphism of $SU(2)$ onto $SO(3)$	74
2.1	The Lie Group $SO(3)$	74
2.2	The Lie Group $SU(2)$	76
2.3	Projection of $SU(2)$ onto $SO(3)$	77
	References	78
	Exercises	79
6	Representations of $SU(2)$ and $SO(3)$	81
1	Irreducible Representations of $\mathfrak{sl}(2, \mathbb{C})$	81
1.1	The Representations \mathcal{D}^j	81
1.2	The Casimir Operator	84
1.3	Hermitian Nature of the Operators J_3 and J^2	84
2	Representations of $SU(2)$	86
2.1	The Representations \mathcal{D}^j	86
2.2	Characters of the Representations \mathcal{D}^j	89
3	Representations of $SO(3)$	90
	References	90
	Exercises	91
7	Spherical Harmonics	93
1	Review of $L^2(S^2)$	93
2	Harmonic Polynomials	94
2.1	Representations of Groups on Function Spaces	94
2.2	Spaces of Harmonic Polynomials	94
2.3	Representations of $SO(3)$ on Spaces of Harmonic Polynomials	95
3	Definition of Spherical Harmonics	97
3.1	Representations of $SO(3)$ on Spaces of Spherical Harmonics	97
3.2	The Casimir Operator	99
3.3	Eigenfunctions of the Casimir Operator	99
3.4	Bases of Spaces of Spherical Harmonics	100
3.5	Explicit Formulas	103
	References	104
	Exercises	104
8	Representations of $SU(3)$ and Quarks	107
1	Review of $\mathfrak{sl}(n, \mathbb{C})$, Representations of $\mathfrak{sl}(3, \mathbb{C})$ and $SU(3)$	107
1.1	Review of $\mathfrak{sl}(n, \mathbb{C})$	107
1.2	The Case of $\mathfrak{sl}(3, \mathbb{C})$	107
1.3	The Bases (I_3, Y) and (I_3, T_8) of \mathfrak{h}	109
1.4	Representations of $\mathfrak{sl}(3, \mathbb{C})$ and of $SU(3)$	110
2	The Adjoint Representation and Roots	110
3	The Fundamental Representation and Its Dual	111
3.1	The Fundamental Representation	111
3.2	The Dual of the Fundamental Representation	112

4	Highest Weight of a Finite-Dimensional Representation.....	113
4.1	Highest Weight	113
4.2	Weights as Linear Combinations of the λ_i	114
4.3	Finite-Dimensional Representations and Weights	115
4.4	Another Example: The Representation 6	116
4.5	One More Example: The Representation 10	117
5	Tensor Products of Representations	118
6	The Eightfold Way	121
6.1	Baryons ($B = 1$)	122
6.2	Mesons ($B = 0$)	122
6.3	Baryon Resonances	123
7	Quarks and Antiquarks	124
	References	125
	Exercises	125
	Problems and Solutions	129
1	Restriction of a Representation to a Finite Group	129
2	The Group $O(2)$	131
3	Representations of the Dihedral and Quaternion Groups	134
4	Representations of $SU(2)$ and of \mathfrak{S}_3	143
5	Pseudo-unitary and Pseudo-orthogonal Groups	147
6	Irreducible Representations of $SU(2) \times SU(2)$	152
7	Projection Operators	160
8	Symmetries of Fullerene Molecules	168
9	Matrix Coefficients and Spherical Harmonics	177
	Bibliography	185
	Index	191