

Contents

Preface	ix
1 Elementary Number Theory	1
1.1 Divisibility	1
1.2 Primes and factorization	6
1.3 Congruences	11
1.4 Solving congruences	15
1.5 Theorems of Fermat and Euler	20
1.6 RSA cryptosystem	28
2 Groups	33
2.1 Definition of a group	33
2.2 Examples of groups	34
2.3 Subgroups	43
2.4 Cosets and Lagrange's Theorem	49
3 Rings	55
3.1 Definition of a ring	55
3.2 Subrings and ideals	61
3.3 Ring homomorphisms	63
3.4 Integral domains	65
4 Fields	67
4.1 Definition and basic properties of a field	67
5 Finite Fields	73
5.1 Number of elements in a finite field	73
5.2 How to construct finite fields	75
5.3 Properties of finite fields	82
5.4 Polynomials over finite fields	86
5.5 Permutation polynomials	89
5.6 Applications	91
5.6.1 Orthogonal Latin squares	91
5.6.2 Diffie/Hellman key exchange	94

6 Vector Spaces	99
6.1 Definition and examples	99
6.2 Basic properties of vector spaces	103
6.3 Subspaces	109
7 Polynomials	111
7.1 Basics	111
7.2 Unique factorization	115
7.3 Polynomials over the real and complex numbers	117
7.4 Root formulas	118
8 Linear Codes	127
8.1 Basics	129
8.2 Hamming codes	132
8.3 Encoding	134
8.4 Decoding	137
8.5 Further study	143
8.6 Exercises	145
9 Appendix	149
9.1 Mathematical induction	149
9.2 Well-ordering Principle	152
9.3 Sets	154
9.4 Functions	158
9.5 Permutations	160
9.6 Matrices	163
9.7 Complex numbers	165
10 Hints and Partial Solutions to Selected Exercises	167
Bibliography	195
Index	199