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POLARIZATION IN SPECTRAL LINES

E. LANDI DEGL'INNOCENTI AND M. LANDOLFI

The scientific research based on spectropolarimetric techniques is undergoing a phase of rapid growth. Instruments of unprecedented sensitivity are nowadays available, particularly for solar observations. To fully exploit the rich diagnostic content of such observations, it is necessary to understand the physical mechanisms involved in the generation and transfer of polarized radiation in astrophysical (or laboratory) plasmas.

After an introductory part based on classical physics, this book tackles the subject by a rigorous quantum-mechanical approach. The transfer equations for polarized radiation and the statistical equilibrium equations for the atomic density matrix are derived directly from the principles of Quantum Electrodynamics. The two sets of equations are then used to present a number of applications, mainly concerning the diagnostics of solar magnetic fields.

This book is primarily addressed to scientists working in the field of spectropolarimetry. It may also serve as a textbook for a course at the graduate or advanced undergraduate level.

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