

# Contents

<b>Preface</b>	<b>xi</b>
<b>1 Conceptual basis of classical mechanics</b>	<b>1</b>
1.1 Newton's three laws . . . . .	2
1.2 Homogeneity and isotropy . . . . .	7
1.3 Solution process . . . . .	9
1.4 Conservative forces . . . . .	11
1.5 Simple harmonic motion . . . . .	12
1.6 Damped and forced oscillator . . . . .	15
1.6.1 Damped oscillator . . . . .	15
1.6.2 Forced oscillator . . . . .	17
1.7 The simple pendulum problem . . . . .	20
1.8 Conservation principles . . . . .	22
1.8.1 Conservation of linear momentum . . . . .	22
1.8.2 Conservation of angular momentum . . . . .	23
1.8.3 Conservation of energy . . . . .	24
1.9 Perturbative analysis and the quartic oscillator . . . . .	28
1.10 Rewriting Newton's second law in terms of kinetic and potential energy in a conservative system . . . . .	30
1.11 Summary . . . . .	32
<b>2 Central force problems</b>	<b>35</b>
2.1 Inertial and gravitational mass: Principle of equivalence . . . . .	36
2.2 Derivation of Kepler's three laws . . . . .	38
2.3 Properties and equations of orbits . . . . .	40
2.4 Integral representations . . . . .	42
2.5 A general class of power law potentials . . . . .	42
2.6 Mapping the general class of potentials: Orbit equation for the inverse square law problem . . . . .	45
2.7 Coulomb and isotropic oscillator potentials . . . . .	45
2.8 Laplace–Runge–Lenz vector . . . . .	49
2.9 Summary . . . . .	51



<b>3</b>	<b>Lagrangian formulation in mechanics</b>	<b>53</b>
3.1	Constraints and generalized coordinates . . . . .	54
3.2	Formulation of D'Alembert's principle . . . . .	58
3.3	Kinetic energy of a holonomic system . . . . .	60
3.4	Lagrange's equations of motion . . . . .	62
3.5	Lagrange's equations for some simple systems . . . . .	66
3.5.1	Plane pendulum . . . . .	66
3.5.2	Spherical pendulum . . . . .	67
3.5.3	Binary star system . . . . .	68
3.5.4	A system with four degrees of freedom . . . . .	69
3.5.5	The problem of a damped oscillator . . . . .	70
3.5.6	A conservative scleronomic system . . . . .	71
3.6	Ignorable coordinates: Routh's procedure of solution . . . . .	72
3.7	Liouville's class of Lagrangians . . . . .	76
3.8	Small oscillations . . . . .	80
3.9	Summary . . . . .	87
<b>4</b>	<b>Hamiltonian and Poisson bracket</b>	<b>91</b>
4.1	The Hamiltonian . . . . .	92
4.2	Hamiltonian canonical equations of motion . . . . .	94
4.3	Poisson bracket . . . . .	101
4.4	Properties of Poisson bracket . . . . .	102
4.5	Poisson theorem . . . . .	104
4.6	Angular momentum . . . . .	105
4.7	Liouville's theorem . . . . .	107
4.8	The case of singular Lagrangians . . . . .	109
4.9	Higher derivative classical systems . . . . .	111
4.10	The Pais-Uhlenbeck oscillator . . . . .	112
4.11	Summary . . . . .	113
<b>5</b>	<b>Dynamical systems: An overview</b>	<b>117</b>
5.1	Basic notions and preliminaries . . . . .	118
5.2	Simple examples from classical mechanics . . . . .	122
5.3	Analysis of a linear system . . . . .	125
5.4	Nonlinear systems: Process of linearization . . . . .	133
5.5	Lotka-Volterra model . . . . .	138
5.6	Stability of solutions: Lyapunov function . . . . .	141
5.7	Van der Pol oscillator and limit cycles . . . . .	145
5.8	Bifurcations . . . . .	151
5.9	Summary . . . . .	158



<b>6</b>	<b>Action principles</b>	<b>161</b>
6.1	The principle of stationary action . . . . .	161
6.2	Corollaries . . . . .	166
6.3	Continuous systems: Uniform string problem . . . . .	169
6.4	Normal modes of oscillation . . . . .	172
6.5	Extended point transformation and $\Delta$ variation . . . . .	174
6.6	$\Delta$ and $\delta$ variations . . . . .	177
6.7	Brachistochrone problem . . . . .	180
6.8	Summary . . . . .	182
<b>7</b>	<b>Motion in noninertial coordinate systems</b>	<b>185</b>
7.1	Rotating frames . . . . .	186
7.1.1	Basic equations . . . . .	186
7.1.2	Some remarks on the Coriolis force . . . . .	190
7.1.3	Effective gravitational constant . . . . .	191
7.1.4	Foucault's pendulum . . . . .	192
7.2	Nonpotential force . . . . .	194
7.3	Summary . . . . .	195
7.4	Examples . . . . .	196
<b>8</b>	<b>Symmetries and conserved quantities</b>	<b>199</b>
8.1	Condition of invariance and Noether's theorem . . . . .	200
8.2	Operator approach . . . . .	205
8.2.1	Symmetry operator . . . . .	206
8.2.2	Parity transformation . . . . .	207
8.2.3	Time-reversal symmetry . . . . .	208
8.3	Virial theorem . . . . .	209
8.4	Summary . . . . .	210
<b>9</b>	<b>Hamilton–Jacobi equation and action-angle variables</b>	<b>213</b>
9.1	Canonical transformation . . . . .	215
9.2	Symplectic property . . . . .	218
9.3	Idea of a generating function . . . . .	221
9.4	Types of time-dependent canonical transformations . . . . .	223
9.4.1	Type I canonical transformation . . . . .	223
9.4.2	Type II canonical transformation . . . . .	224
9.4.3	Type III canonical transformation . . . . .	225
9.4.4	Type IV canonical transformation . . . . .	225
9.5	Infinitesimal canonical transformations . . . . .	228
9.6	Hamilton–Jacobi equation . . . . .	229



9.6.1	Time independent Hamilton–Jacobi equation: Hamilton’s characteristic function . . . . .	232
9.6.2	Other variants of Hamilton–Jacobi equation . . . . .	235
9.7	Action-angle variables . . . . .	235
9.7.1	Motion of a particle in a 2-dimensional rectangular well . . . . .	242
9.8	Possible trajectories . . . . .	243
9.8.1	Periodic trajectories . . . . .	243
9.8.1.1	Some explicit examples for periodic trajectories . . . . .	244
9.8.2	Open trajectories . . . . .	247
9.8.3	Special trajectories when the billiard ball hits a corner . . . . .	247
9.9	Summary . . . . .	248
	<b>References</b>	<b>253</b>
	<b>Index</b>	<b>257</b>