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When teaching this material to both computer science and mathematics students, I have found that the book can be a valuable resource. The field of computational geometry is a rapidly growing area of research, and this book provides a comprehensive overview of the field. The book is divided into several parts, each covering a different aspect of the field. The first part covers the basics of computational geometry, including the definition of the field and the basic algorithms. The second part covers more advanced topics, such as the theory of convex polytopes and the theory of motion planning. The third part covers applications of computational geometry, such as in computer graphics and robotics. The book is written in a clear and concise style, and it includes many examples and exercises. I highly recommend this book to anyone interested in computational geometry.

**Topics Covered**

I consider the "core" concerns of computational geometry to be polygon partitioning (including triangulation), convex hulls, Voronoi diagrams, arrangements of lines, geo-metric searching, and motion planning. These topics form the backbone of the book. The field is not so settled that this list can be considered a consensus; other researchers would define the core differently.

Many textbooks include far more material than can be covered in one semester. This is not such a text. I usually cover about 80% of the text with undergraduates in one 40 class-hour semester and all of the text with graduate students. In order to touch on each of the core topics, I find it necessary to oscillate the level of detail, only reaching some algorithms while detailing others. Which ones are sketched and which detailed is a personal choice that I can only justify by my classroom experiences.

- Multitask robot arm reachability

Researchers in industry find for working code for their favorite algorithms. The material in this text should be accessible to students with only minimal preparation. Discrete mathematics, calculus, and linear algebra suffice for mathematics. In fact, very few students find this book for working code for their favorite algorithms.

<sup>1</sup>E.g., Hoffmann (1989) and Morisson (1990).  
<sup>2</sup>The distribution also includes the code for the cube (Figure 4.14), random points on a sphere (Figure 4.15), and the book cover image.  
<sup>3</sup>Exercise 5.5.5 [12].