

Morphometrics is the statistical study of biological shape and shape change. Its richest data are landmarks, points such as "the bridge and the nose," that have biological names as well as geometric locations. This book is the first systematic survey of morphometric methods for landmark data. The methods presented here combine conventional multivariate statistical analysis with themes from plane and solid geometry and from biomathematics to support biological insights into the features of many different organs and organisms.

The book begins with a review of the fundamentals of landmarks and a discussion of the thin-plate spline, a new, statistically tractable implementation of the old model of shape change as deformation. This is followed by a critical survey of conventional multivariate morphometrics (the use of interlandmark distances as separate variables). Coordinates for representing landmark configurations without reference to size are then introduced, and their multivariate statistics explored in full.

The second half of the book is a survey of the most general and powerful new methods for describing the results of these analyses for both simple and complex landmark configurations. Using diagrams linked to biological interpretation, the text explains and interrelates the geometric features by which morphometric findings can be rendered legible. New tools are demonstrated using a variety of data sets from evolutionary biology, micropaleontology, neuroanatomy, and craniofacial growth.

This book should be of value to applied statisticians and geometers, as well as to all biological and biomedical researchers who need quantitative analyses of information from biomedical images.

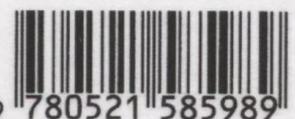
"...will certainly be a landmark volume in this difficult but important field of shape analysis."

-K. V. Mardia, *ISI Short Book Reviews*

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- 1 Introduction** 1
- Morphometrics is the study of covariances of biological form.
- 1.1 Four principles** 1
- Four principles underlie the morphometrics of landmark data: (1) archiving biological form by locations of landmark points, (2) converting sets of three locations to pairs of shape coordinates, (3) processing these variables by carefully contrived multivariate statistical maneuvers, and (4) interpreting findings in the picture plane or space of the data.
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6.4 Analyses of more than three landmarks

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6.5 Biometric analysis of triangles of landmarks: examples

The methods just introduced are demonstrated in a diversity of data sets. Exemplary analyses are shown, and their interpretations explored, for group mean differences in form, in growth, and in growth allometry; for the correlation of form with exogenous factors; and for the directionality of digitizing noise.

6.6 A comment on “finite elements”

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