CONTENTS

Chapter 1. Introduction 1
1.1. What is model theory? 1
1.2. Model theory for sentential logic 4
1.3. Languages, models and satisfaction 18
1.4. Theories and examples of theories 36
1.5. Elimination of quantifiers 49

Chapter 2. Models constructed from constants 61
2.1. Completeness and compactness 61
2.2. Refinements of the method. Omitting types and interpolation theorems 77
2.3. Countable models of complete theories 96
2.4. Recursively saturated models 109
2.5. Lindström's characterization of first order logic 127

Chapter 3. Further model-theoretic constructions 136
3.1. Elementary extensions and elementary chains 136
3.2. Applications of elementary chains 147
3.3. Skolem functions and indiscernibles 163
3.4. Some examples 178
3.5. Model completeness 186

Chapter 4. Ultraproducts 211
4.1. The fundamental theorem 211
4.2. Measurable cardinals 227
4.3. Regular ultrapowers 248
4.4. Nonstandard universes 262

Chapter 5. Saturated and special models 292
5.1. Saturated and special models 292
5.2. Preservation theorems 306
5.3. Applications of special models to the theory of definability . 323
5.4. Applications to field theory 342
5.5. Application to Boolean algebras 372

Chapter 6. More about ultraproducts and generalizations . . 384
6.1. Ultraproducts which are saturated 384
6.2. Direct products, reduced products, and Horn sentences . 405
6.3. Direct products, reduced products, and Horn sentences (continued) 420
6.4. Limit ultrapowers and complete extensions . . . 447
6.5. Iterated ultrapowers 463

Chapter 7. Selected topics 482
7.1. Categoricity in power 482
7.2. An extension of Ramsey's theorem and applications; some two-cardinal theorems 509
7.3. Models of large cardinality 534
7.4. Large cardinals and the constructible universe . . . 558

Appendix A. Set theory 579

Appendix B. Open problems in classical model theory . . . 597

Historical notes 603

References 623

Additional references 634

Index of definitions 641

Index of symbols 649