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The following identify the major symbols and notation used in the text. To the greatest extent possible, we have attempted to keep unique meanings for each item. In some cases where we use the additional item, they should be clear from context. The symbols are only used in more than one section. Additional notation may be needed within specific sections and is explained when used.

Row and column vectors are denoted to be column vectors to indicate row vectors. The scalar product of two n -vectors, x and y , is denoted by $x^T y$ for first derivatives with respect to time (e.g., $\dot{y} = dy/dt$).

Linear and nonlinear programs are represented by lowercase Latin letters while matrices are uppercase. Most variables and certain scalars are generally Greek letters. Subscripts indicate a stage while subscripts indicate components followed by reference index. Stochastic variables and random quantities, expectations of random variables are indicated by a bar (\bar{x} , μ , or $E(x)$). We also use the bar notation to indicate the mean value of a random variable.

References are given in the text by section and number within the chapter (e.g., Section 3, Equation 2). For references to chapters other than the current one, we use three indices: chapter, section, and equation, (e.g., Section 2, Chapter 3, Section 1, Equation 2). Exercises are given at the end of chapters for reinforcement at the end of Sections 3.2 and 3.4 and are referenced by the page number as equations. All other items (figures, tables, definitions, examples) are located consecutively through the entire chapter with a single reference (e.g., Figure 1.1 within the current chapter and chapter and number if in a different chapter, e.g., Figure 3.1 for Chapter 3, Figure 1).