

# *Springer Series in Statistics*

Joseph G. Ibrahim, Ming-Hui Chen, and Debajyoti Sinha

## Bayesian Survival Analysis

Survival analysis arises in many fields of study including: medicine, biology, engineering, public health, epidemiology, and economics. This book provides a comprehensive treatment of Bayesian survival analysis. Several topics are addressed, including: parametric models, semiparametric models based on prior processes, proportional and nonproportional hazards models, frailty models, cure rate models, model selection and comparison, joint models for longitudinal and survival data, models with time varying covariates, missing covariate data, design and monitoring of clinical trials, accelerated failure time models, models for multivariate survival data, and special types of hierarchical survival models. Also, various censoring schemes are examined including right and interval censored data. Several additional topics are discussed, including: noninformative and informative prior specifications, computing posterior quantities of interest, Bayesian hypothesis testing, variable selection, model selection with nonnested models, model checking techniques using Bayesian diagnostic methods, and Markov chain Monte Carlo (MCMC) algorithms for sampling from the posterior and predictive distributions.

The book presents a balance between theory and applications, and for each class of models discussed, detailed examples and analyses from case studies are presented whenever possible. The applications are all essentially from the health sciences, including cancer, AIDS, and the environment. It would be most suitable for second- or third-year graduate students in statistics or biostatistics. It would also serve as a useful reference book for applied or theoretical researchers as well as practitioners.

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